# The Effect of Stricter Capital Regulation on Banks' Risk-Taking: Theory and Evidence*

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**Abstract**

When a bank's capital is constrained by regulation, regulatory cost (risk weightings) alters the risk and value calculations for the bank's assets. A simple portfolio choice model shows how banks' asset portfolios are affected by regulation and also points to other determinants of banks' asset portfolios. In particular, we find that banks may respond to stricter regulation by *increasing* the share of high-risk assets. We make an empirical examination of U.S. banks' asset portfolios in the period from 2002 to 2014. Our results show that banks responded to the implementation of the stricter Basel II regulations by *increasing* the share of high-risk assets in the risky part of their portfolios. In addition, we find evidence of portfolio inertia.

*Keywords:* Banks; asset risk; credit risk; portfolio choice; risk-based capital regulation

*JEL classifications:* G11; G18; G21; G28

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## 1. Introduction

The recent financial crisis, especially the sub-prime mortgage crisis, has placed a sharp spotlight on banks' risk taking. In the media, banks are often blamed for shrugging off risk concerns while pursuing higher earnings for example through high leverage or through granting loans with high credit risk. Indeed, in a model with just enough frictions for banks to have a meaningful role in producing socially valuable liquid claims, DeAngelo and Stulz (2015) find that banks will optimally be highly leveraged. However, the concern is that this type of behavior on part of the banks will leave the financial system vulnerable to economy-wide shocks. Regulatory frameworks, such as the Basel Accords[1] have been put in place to counteract such tendencies among banks, initially by putting an upper limit on banks' risk-taking and altering their cost-benefit analyses.

Since a simple flat ratio of capital to asset might incentivize banks to hold more high-risk assets (see, e.g., Koehn and Santomero, 1980; Kim and Santomero, 1988), regulators have been refining capital regulation to match the actual risk of banks' assets. Yet, Basel II has been questioned for exacerbating procyclicality of banks' lending (see, e.g., Repullo and Suarez, 2013; Behn et al., 2016). In response to this type of criticism, Basel III adds a capital preservation buffer and a countercyclical buffer, and Basel IV emphasizes the calculation of the risk-weighted assets[2] and reconciles the internal ratings-based approach with the standardized approach.

[1]The Basel Accords are the supervision accords for banks promulgated by the Basel Committee on Banking Supervision. This study limits its focus to the aspects of the Accords that address capital adequacy, which is at the center of the Accords.

[2]To value the overall risk of a bank's assets, the Basel Accords use total risk-weighted assets, where a higher weight is assigned to assets with higher risk. Under Basel I and the standard approach of Basel II, there are four broad categories of risk: 0%, 20%, 50%, and 100% risk weightings. The risk under Basel I is mainly credit risk. To determine capital adequacy, the Basel Accords use a risk-based capital ratio: the ratio of total regulatory capital to total risk-weighted assets. Under Basel I (1998) and II (2004), a bank has to reserve total capital equal to at least 8% of the value of the bank's total risk-weighted assets; under Basel III (2010), a bank has to hold additional conservation and countercyclical buffers.

How do individual banks navigate in a landscape of cycles of credit yields and risk such as during the pre-crisis surge in yields in the sub-prime markets or the credit shocks induced by the failure of Lehman Brothers, with simultaneous changes in capital regulation? For instance, if a bank faces increasing default probabilities and default correlations among its clients, how does it reassess the credit risks of existing and new potential assets and decide on a reallocation, while also facing a more stringent capital regulation? To answer these questions, we revisit the literature on banks' asset portfolio choices (Koehn and Santomero, 1980; Kim and Santomero, 1988; Rochet, 1992; Furfine, 2001; Milne, 2002) with a special focus on credit risk.

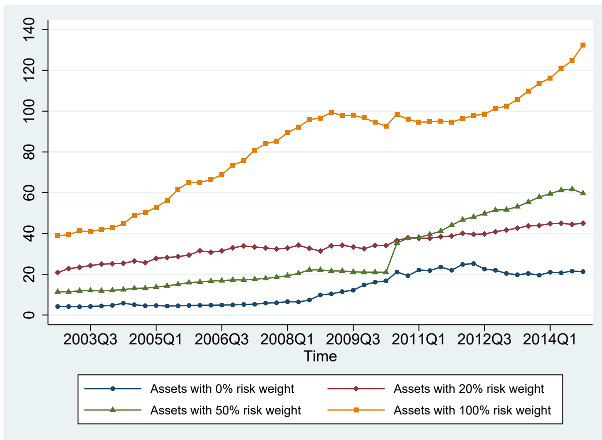
First, we look into US banks'[3] total assets[4] with different levels of risk, defined by the risk weightings under the Basel Accords. Figures 1a and 1b display the sums of the assets with certain risk weightings for all U.S. banks from the fourth quarter of 2002 to the fourth quarter of 2014. According to Figure 1a, there is a distinct increase in the amount of assets with the highest risk (100% risk weighting), although the trend for its proportion of the total is not obvious (Figure 1b). Figure 2 shows banks' allocation among risky assets whose risk weightings are nonzero. Prior to the financial crisis, there is an increase in the proportion allocated to the riskiest assets. Since it takes some time to adjust long-term assets, this proportion declines sometime after the onset of the financial crisis, but when it declines, we see a sharp decline. However, towards the end of the sample period, the banks' risk-taking starts to increase again. Combining Figures 1 and 2 with common knowledge regarding business cycles, we see that banks' risk-taking tends to increase during upturns,

[3]Here, and in the rest of the paper, we refer to stand-alone bank-holding companies and stand-alone commercial banks as "banks". With this strict definition, we avoid considering potentially misleading data emerging from regulatory arbitrage within financial conglomerates.

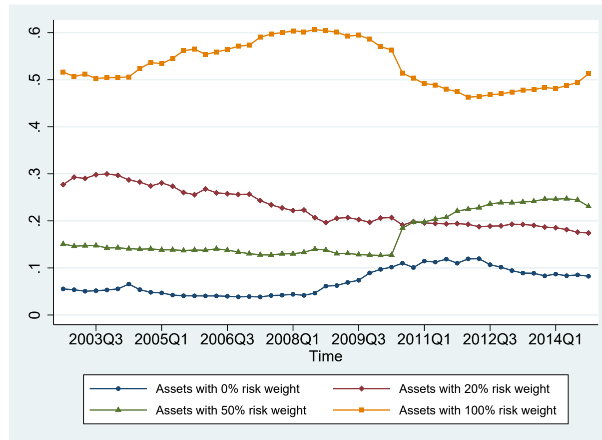
[4]For assets with 0% risk weight, total amounts are not available in CapitalIQ, which is the data source of our empirical tests. Here, we extract the data on assets within different risk categories for the banks, in our sample in the empirical tests, from the regulatory reports available from the databases of the Federal Reserve.

**Figure 1:** Banks' allocation to assets with different risk weightings

**(a)** Allocation to different asset classes




**(b)** Allocation to different asset classes in proportion




This figure displays total asset amounts within different risk categories of all the stand-alone banks and bank-holding companies in the U.S., measured in trillions of dollars in Figure 1a and in proportions to total amounts in Figure 1b

i.e., they exhibit procyclicality, a stylized fact that has been established in the previous literature (e.g., Berger and Udell, 2004; Santos and Winton, 2008; Murfin, 2012). Recent bank regulation (in particular Basel III & IV) intends to counteract this phenomenon.[5]

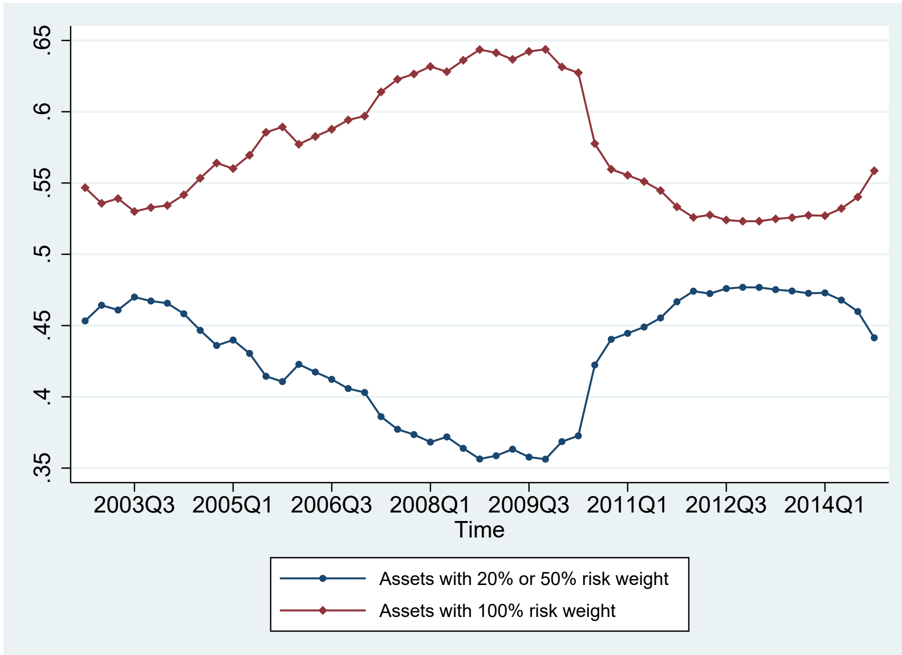
Basel II was revised to closely match banks' actual asset risk and we can also regard it as a tightened regulation compared to Basel I, since there is less room for regulatory arbitrage. The risk-based capital-adequacy requirements pose additional costs for riskier assets, since banks have to reserve more capital for assets with a higher credit risk. How do banks react to such regulatory changes? Banks also have incentives to take more risk in order to gain higher earnings and compensate for the higher costs of their capital reserves. Thus, whether or not a tightening capital requirement would have the desired effect is an open question.

To address the above-mentioned questions, we regard a bank as its assets'[6] manager and consider

[5]Notice that the figures reflect mainly credit risk before the second quarter in 2008, when Basel II took effect.

[6]Banks hold different types of assets, such as loans and securities. Our model focuses on credit risk, which is the central risk facing commercial banks and also the main concern of capital regulation.

**Figure 2:** Banks' allocation among assets with nonzero risk weightings




This figure shows the proportions of high- and low-risk assets to total amounts allocated to risky assets, i.e. proportions of assets with 100% risk weight and proportions of assets with 20% and 50% risk weights, respectively, for all stand-alone banks and bank-holding companies in the U.S.

her portfolio allocation with a minimum regulatory capital requirement as a possible binding condition. Since our focus is on banks' risk-taking and asset allocation, we deliberately abstract from banks' interest setting behavior, screening and monitoring – in line with, e.g., Altman and Saunders (1998), Kealhofer and Bohn (2001), and Mencía (2012). We find that when a bank's capital is not constrained by regulation, its asset allocation decision depends on the risk measure of assets – namely, the cash-flow volatility around the expected loss due to default risk – and on the key measure of an asset's valuation, the Sharpe ratio (Sharpe, 1966), modified according to our settings. However, when the bank's capital is constrained by regulation, regulatory cost (risk weighting in risk-based capital regulation) steps in and weights the cash-flow volatility, and even replaces the volatility in the measure of the assets' valuation (reward-to-regulatory-cost ratio instead of Sharpe's reward-to-variability ratio). If the regulator imposes a new and more stringent regulation banks might decrease or increase their risk exposure within their risky funds of assets depending on these reward-to-regulatory-cost ratios.

We test these implications using bank-level data on assets with different risk categories for all US banks. Since it takes years to formally implement each set of Basel rules, we focus on the shift from Basel I to Basel II. Moreover, since detailed information of assets in each risk category and consequently their credit risk is not available, we extract macro-level credit yields, default probabilities and default correlation based on corporates' credit ratings, used in the standardized approach under Basel II. The empirical examination largely verifies our predictions of how banks' choices between high-risk, high-earning assets and low-risk, low-earning assets react to the updated information on assets' earnings and default probabilities, and we find that the implementation of a stricter regulation through the introduction of Basel II actually led them to *increase* the share of high-risk assets in the risky part of their portfolios. Further, our results suggest that banks' asset portfolio decisions are positively related to previous decisions and thus exhibit inertia.

Although there are theory models evaluating portfolio credit risk, only a few articles concern credit-portfolio optimization (see, e.g., Altman and Saunders, 1998; Kealhofer and Bohn, 2001; Mencía, 2012). Regarding the impact of capital regulation on banks' asset risk, the theoretical literature yields mixed predictions, with a few studies from the point of view of portfolio management (see, e.g., Koehn and Santomero, 1980; Kim and Santomero, 1988; Rochet, 1992; Furfine, 2001; Milne, 2002). Our paper mainly contributes to the literature on banks' risk taking by analyzing banks' asset allocation explicitly with respect to credit risk, by disentangling the effects of risk-based capital regulation on banks' asset risk, and by performing an empirical investigation of the dynamics of banks' asset allocation when facing macro-level cycles in credit risk.

The remainder of the paper is organized as follows. Section 2 reviews the literature. In Section

3, we develop our hypotheses. In Section 4, we examine our hypotheses empirically using a panel data set and in the process, we include details on how we estimate conditional default probabilities, default correlation, and payoffs. Section 5 concludes.

## 2. Literature review

Over the past two decades, we have seen important advances in the modeling of correlated defaults and the evaluation of portfolio credit risk, including Moody's KMV *Portfolio Manager*, JPMorgan's *CreditMetrics*, Credit Suisse's *CreditRisk*$^+$, McKinsey's *CreditPortfolioView*, correlated default-intensity models, and copula-based modeling.[7]

Yet, there are only a few articles on credit-portfolio optimization. In addition to reviewing the literature, Altman and Saunders (1998) propose a new measure of the return-risk tradeoff, where they measure a portfolio's risk by its unexpected loss, determined by the standard deviation around the expected loss, which is estimated historically over time using bond rating equivalence: the $Z''$-score (Altman, 1993). Similarly, in a technical report from KMV (Kealhofer and Bohn, 2001), they measure unexpected loss by the standard deviation of loss only due to default in a default-only model, where there are two states; default and no default. Mencía (2012) models homogeneous loan classes, each comprising conditional independent loans whose conditional default probability is a probit function of a Gaussian state variable. He shows that, in his setting, mean-variance analysis is fully consistent with CRRA utility maximization. All three articles mentioned above adapt the mean-variance framework (Markowitz, 1952), to analyze the risk and returns on credit portfolios.

Regarding the impact of capital regulation on banks' asset risk, the theoretical literature yields

[7]See detailed descriptions of the models by Gordy (2000), Crouhy et al. (2000), and Duffie and Singleton (2003), among others.

mixed predictions, although there is general agreement about the immediate effects of stricter capital requirements on total bank lending and the longer-term impact on capital ratios. The immediate effects of stricter capital requirements are reduction in total lending, increases in market loan rates and substitution away from lending to alternative assets. In the long-term, stricter capital requirements have been found to lead to an increase in capital ratios. However, there are largely divergent conclusions in the previous literature regarding how capital regulation influences individual banks' choices on the margin (see VanHoose, 2007, among others). As yet, there are just a few studies of banks' asset risk from the point of view of portfolio management. Koehn and Santomero (1980) and Kim and Santomero (1988) consider a mean-variance portfolio-selection model, showing that a higher uniform regulatory capital ratio constrains the efficient asset investment frontier and might actually result in a higher asset risk and increase banks' insolvency risk, yielding the opposite of the intended effect. Nevertheless, Kim and Santomero (1988) model the optimal weights for the risk-based capital requirement, and predict that, with higher weights for riskier assets, banks would hold more liquid safe assets and fewer risky assets. Rochet (1992) argues that, if banks behave as portfolio managers – maximizing utility instead of the market value of their future profits as in Furlong and Keeley (1989), among others – capital regulation can be effective, but only if the risk weights are proportional to the systematic risks of the assets (their betas). Furfine (2001), developing a dynamic value-maximizing model and calibrating it to U.S. data, finds that Basel I was involved in the credit crunch experienced in the 1990s and predicts that, under Basel II, banks would increase loans relative to securities and safer loans relative to risky ones. Milne (2002) interprets capital regulation as a system of sanctions for ex post violation instead of ex ante enforcement, and his value-maximizing model suggests that there is relatively less need to match risk weightings accurately to portfolio risk.

Based on the framework developed by Koehn and Santomero (1980) and Kim and Santomero (1988) we set up a portfolio-selection model that allows us to study the effects of risk-based capital regulation explicitly. However, in our model, the bank manager maximizes the utility of one-period net value of assets, instead of the utility of equity returns, as in their models. The obvious advantage of using the utility of assets' net value is its focus on the bank's asset risk, which we believe is closer to established practice.

The most marked difference from the aforementioned banking literature is that this paper focuses on credit risk, where there are only two conditional states: default and no default. Thus, we can adapt similar approaches from the credit-portfolio-optimization literature (Altman and Saunders, 1998; Kealhofer and Bohn, 2001; Mencía, 2012). Moreover, we study whether banks restructure their portfolios from low-risk, low-earning assets to high-risk, high-earning assets to compensate for additional costs imposed by capital requirements.

## 3. Hypotheses development

In order to fix ideas, we develop a single-period, three-asset (risk free, low risk, high risk) model of banks' portfolio allocation. All of the details of this model are in Appendix A. In this section, we describe this model and its testable implications.

We model a bank as its asset manager, who makes one-period decisions on allocating resources (deposits and capital) for the assets with different levels of credit risk; its capital might be constrained by risk-based capital regulation. The model predicts how the bank manager restructures the portfolio of different assets when their conditional default probabilities, default correlation, or payoffs change, or when the regulator tightens risk-based capital requirements. Since we focus on banks' risk-taking, we deliberately abstract from banks' interest rate setting behavior, screening and monitoring.

The bank aims to maximize a single-period expected quadratic utility of its assets' random cash profit, and the expected utility is an increasing function of the expected cash flow and a decreasing function of the cash-flow variance.[8] The bank chooses among three types of asset: a high-risk, high-earning asset; a low-risk, low-earning asset; and a risk-free asset.[9] For simplicity, only the relative sizes of assets are assumed to be under the control of bank management.[10] In addition, the regulator decides that the bank has to hold capital as minimum $k$ times the total risk-weighted assets, where risky assets are assigned higher weights. It is also assumed that the holding period perfectly matches the maturity of the assets. All assets are in perfectly elastic supply, i.e., the bank is a price taker. Liabilities, capital and deposits are exogenous, so this is a pure asset management problem (no liability management) to which standard quadratic utility (CAPM) results apply.

Absent of capital requirements, the portfolio allocation depends on a modified Sharpe ratio, depending on default risk and cash flow volatility (Equations A.9–A.11). With binding capital requirements, however, there is a bias either away from or towards high-risk assets. Which of these two possibilities arises depends on the relative ratio of returns and regulatory capital costs for the two types of asset (Equations A.21–A.23).

If there are no capital constraints, the usual two-fund separation theorem applies (Tobin, 1958)

[8]Bawa and Lindenberg (1977) show that, as long as the expected utility can be written as an increasing function of the expected return and a decreasing function of the variance of the portfolio only, without any assumption of probability distributions of assets' returns, the optimal portfolio lies on the efficient frontier in the mean-variance framework of Markowitz (1952). Also, when there is a risk-free asset, the two-fund theorem is valid. Here, by applying a quadratic utility function, we could sidestep most of the problems associated with solving a general utility-based portfolio choice and obtain an analytical solution.

[9]The choice of three types of assets is also consistent with the empirical examination in Section 4. In addition, the model with four types of assets, which corresponds to the four risk categories of assets in the Basel Accords, is qualitatively identical.

[10]This simplification serves the purpose of this study. While we could enrich the model with additional features, such as variations in the bank's liabilities and capital, how the bank allocates among assets with different credit risks in a *ceteris paribus* environment is not altered.

so the portfolio of risky assets maximizes the Sharpe ratio. As capital requirements are introduced and become binding, we find that there is a shift in asset composition, depending on the parameter

$$\vartheta \equiv \frac{\vartheta_h}{\vartheta_l} = \frac{(\mu_h - r_f)W_l}{(\mu_l - r_f)W_h}, \tag{1}$$

where $\mu_h$ and $\mu_l$ are the expected returns of the high-risk and low-risk assets, respectively, and $W_h$ and $W_l$ are the corresponding risk weights, whereas $r_f$ is the risk-free rate. Further, $\vartheta_h$ and $\vartheta_l$ are the reward-to-regulatory cost ratios, defined as $\vartheta_h = (\mu_h - r_f)/W_h$ and $\vartheta_l = (\mu_l - r_f)/W_l$, respectively (Proposition 3 in the appendix). The $\vartheta$ parameter captures the relative importance of capital requirements and the excess return for the two risky assets.

A binding capital requirement has two effects. First, it limits the ability to accept risk, resulting in a portfolio shift towards the risk-free fund. Second, there might be a reallocation of the risky fund, either towards the high risk asset or the low risk asset. This latter reallocation effect might offset the risk reduction resulting from the shift towards the risk-free fund. The adjustment of the risky fund depends crucially on the parameter $\vartheta$ (see Proposition 3a in the appendix).

If $\vartheta = 1$, then adjustment of the risky fund would lead to a detoriation in risk-return tradeoff (the portfolio Sharpe ratio) without any compensating loosening of the capital constraint. Thus, in this case, there is no adjustment within the risky fund, and the two-fund separation theorem continues to apply.

If $\vartheta > 1$, then the bank can loosen a binding capital constraint (at the cost of worsening the Sharpe ratio) through shifting the risky fund towards the more capital-efficient high-risk asset. Conversely, if $\vartheta < 1$, then the bank can achieve a loosening of the binding capital constraint by the opposite portfolio shift, into the more capital-efficient low-risk asset. In both of these cases, the usual two-fund separation result no longer holds: the investment in the risk-free asset can be less than for the case when $\vartheta = 1$.

In addition, Proposition 2 shows the effects of monetary policy on banks' risk-taking: according to the proposition, an environment with a lower risk-free rate results in higher risk-taking among banks.

Given this theoretical background, we aim at testing whether the variables that are identified as important for portfolio choice in Proposition 1 in the appendix are significant and carry the predicted signs. Also, given the ambiguous answers provided by our reasoning as well as the previous literature, another objective is to investigate whether banks' risk taking increases or decreases in response to stricter regulation. Further, while the model presented in the appendix is a static one, we are also interested in studying the dynamics of banks' portfolio decisions over time.

## 4. Empirical examination

This section tests the model on U.S. stand-alone commercial banks and bank-holding companies. The sample is comprised of 1721 banks with quarterly consolidated bank-level data from the first quarter of 2002 to the fourth quarter of 2014. Due to concerns regarding domestic and international competitiveness, the implementation of capital regulation in the U.S. closely follows the Basel Accords.[11]

Since detailed information on each bank's assets with certain risk weighting – 0%, 20%, 50%, or 100% – is not available due to business confidentiality, we use macro-level data on the corporations with external ratings corresponding to assets' risk weighting according to Basel II to assess assets

[11] General risk-based capital rules based on Basel I have been implemented since 1989; the standardized approach for general banking organizations and the advanced internal ratings-based approach for core banks, based on Basel II, have been implemented since 2008. Core banks are those with consolidated total assets of $250 billion or more or with a consolidated total on-balance-sheet foreign exposure of $10 billion or more (Treasury, the Federal Reserve System, and Federal Deposit Insurance Corporation, 2007, 2008).

within each risk category. Thus, the credit risk and payoff of the bonds issued by those corporations are used to *proxy* the credit risk and payoffs of each bank's assets within one risk category.[12] In terms of statistical tests, we first investigate whether banks absorb the market-wide macro information on credit risk in their decision making on asset allocations as suggested by the model. Second, we test the impact of a tightening capital regulation on banks' asset allocations. Finally, we examine banks' portfolio inertia.

### *4.1. Data*

All financial-statement variables are drawn from quarterly filings of commercial banks' or bank holding companies' reports to the Federal Reserve for regulatory purpose. All data are collected from Standard & Poor's Capital IQ database, unless otherwise stated.

For macro-level estimates of assets' credit risk and payoff, we use Standard & Poor's long-term corporate ratings, the data on the corporations' actual defaults, and yields of bonds issued, extracted from the Capital IQ database. The sample is comprised of all issuers of senior unsecured corporate debentures in the market. Thereafter, we estimate the default probability of each group of corporations with certain ratings, and the default correlation with another group of corporations holding other ratings. To value the macro-level payoff of assets, we use average yield[13] of those bonds issued by the corporations holding certain ratings.

[12]Note that we do not focus on banks' bond allocations; we merely use credit risk and payoffs of corporate bonds within each risk category to proxy for the credit risk and payoffs of all assets (mainly loans) in that risk category.

[13]Since the yield on a bond already accounts for its associated risk, using yield would underestimate the payoff of a type of asset. Nevertheless, the average yield in the market provides a macro level (actually a macro low bound) of the average payoff of the type of asset, which is comparable across time.

### 4.2. Asset categories

Under the standardized approach in Basel Accord II, assets are classified into different risk categories according to their external ratings when the ratings are applicable. Based on Basel Accord II and the requirements for Call Reports, the ratings corresponding to assets with 20%, 50%, and 100% risk are AAA to AA, A, and BBB to BB,[14] respectively.

Yet, there are so few observations of defaults for corporations holding ratings AAA to AA, or A, that the result is a zero default rate in much of the sample period. Therefore, we combine assets with 20% and 50% risk, and assign them an average risk level of 35%. Consequently, there are three types of asset in the sample: risk-free assets, low-risk, low-earning assets, and high-risk, high-earning assets, with 0%, 35%, and 100% risk, respectively, consistent with the model in Section 3. Hence, the credit qualities of low- and high-risk assets are proxied by the market-wide bond issuers holding ratings AAA to A and BBB to BB, respectively.

### 4.3. Estimating default probability, default correlation, and payoff

We estimate the probabilities of default by empirical average cumulative default rates for a historical time period, as is commonly done by the major rating agencies. These historical default rates, based on issuer, give equal weights to all issuers in the calculation, regardless of differences in the nominal size of the bonds issued by each issuer.[15] This approach is also cohort based, which tracks the default rates of firms with a certain rating on a given calendar date, and this pool of issuers

[14]According to the instructions for Call Reports, only the ratings above B are eligible for the ratings-based approach. Although, in accordance with the Dodd–Frank Wall Street Reform and Consumer Protection Act, the U.S. rules do not reference external credit ratings from 2010, in practice U.S. banks often use external ratings: see *Regulatory Consistency Assessment Programme (RCAP), Assessment of Basel III regulations – United States of America*, available at www.bis.org/bcbs/publ/d301.pdf.

[15]We use average default information on issuers instead of issues to obtain the low bound of the macro level of default, since the ratings of issues are generally not higher than that of their issuer.

is a *cohort*. We adopt the method of calculating average cumulative default rates with adjustment for rating withdrawals used by Moody's, as demonstrated by Cantor and Hamilton (2007). We then modify their methodology to estimate the default correlations. The methods are described in Appendix B.

Since there are relatively more default observations on a quarterly basis than on a monthly basis, especially for investment-grade corporations, we employ quarterly cohort spacing, which also produces more accurate estimates of default correlations. For the same reason, we also choose a longer investment horizon of four years.[16] We assume that, when banks' managers make portfolio asset choices, they hold expectations on default probabilities and default correlations based on the historical information during the previous four years.

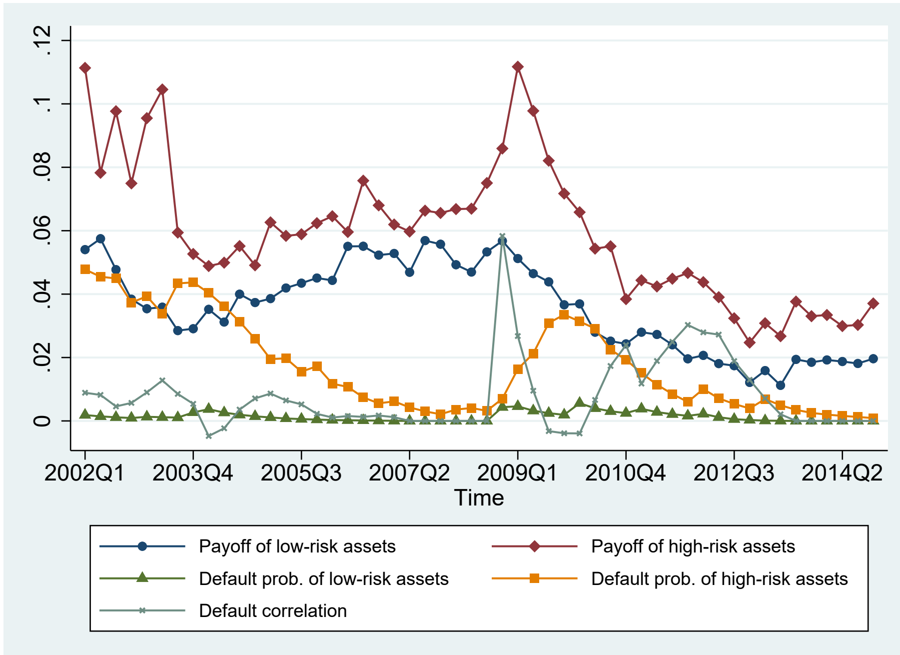
To value the credit risk and payoffs of the assets with a certain risk type, and to preserve the creditworthiness of issuers, we only employ senior unsecured straight bonds: fixed-rate, U.S.-dollar bonds without any asset-backing or credit-enhancement – e.g., callability, puttability, sinking, or convertibility. We then estimate the payoff of each asset type at a date by an average of four-year-to-maturity yields[17] at that date on all available straight bonds whose issuers hold certain ratings.

Figure 3 shows our estimates of macro-level information of credit risk, namely yield, default probability and default correlations. These estimates are used to proxy payoffs and default probabilities of high- and low-risk assets and their default correlation. The figure shows evidence of cyclicality in credit risk. The trends in payoffs and default probabilities of high-risk assets mimic those in

[16]We perform robustness checks on two-year and three-year default rates and correlations: the results are qualitatively similar. The maximum possible length of the estimation window is four years because the data on actual defaults date back to 1998.

[17]The yield that represents one issuer is an average of yields on all available straight bonds issued by that corporation.

**Figure 3:** Macro-level estimates of credit risk and payoffs




*Payoff*, *Default prob.*, and *Default correlation* are estimated based on average macro credit information on the risky assets. The estimated payoffs, default probabilities, and default correlation (in percentage) are calculated at the beginning of each quarter.

allocations to high- and low- risk assets in Figure 2.

### *4.4. CAMELS variables*

We use *CAMELS* variables as controls in our regressions. *CAMELS* is a ratings system used by, e.g., U.S. banking supervisory authorities to assess a bank's overall condition (see Duchin and Sosyura, 2012; Li, 2013, among others). The letters in the acronym stand for *Capital adequacy*, *Asset quality*, *Management capability*, *Earnings*, *Liquidity* and *Sensitivity to market risk*, respectively, and we include proxies to quantify these concepts in our regressions.

Specifically, to proxy *Capital adequacy*, we use the logarithm of the ratio of total equity to total assets and we call this variable ”Equity ratio.” We measure *Asset quality* by using nonperforming assets over total assets and we term this variable ”Nonperforming assets.” To proxy for *Management* *capability*, we calculate total expenditures to total income which, in a sense, is a measure of operating inefficiency; an exception in which it would not be an appropriate measure would be if the aforementioned ratio is negative due to a negative income and thus we treat these cases as missing values. However, if this ratio is negative due to negative expenditures, it still aligns well with the other values, and thus we include such numbers in our empirical analysis. In order to capture *Earnings*, we simply use the return on assets (ROA). Further, *Liquidity* is measured as the ratio of liquid assets to total assets, where ”liquid assets” is automatically calculated by Capital IQ, and we call the aforementioned *ratio* ”Liquid assets” in the regressions. Finally, in line with previous studies, we use the ratio of the absolute value of noninterest income to the sums of the absolute values of noninterest and interest income to capture *Sensitivity to market risk*, and we term this variable ”Noninterest income.”

### *4.5. Results*

Table 1 summarizes statistics of the data on the variables used in the empirical tests. *Proportion of high-risk assets* and *Proportion of low-risk assets* are the banks' actual shares of high-risk (100% risk) and low-risk (20% and 50% risk) assets among the risky (20%, 50%, and 100% risk) assets, respectively. Thus, they sum to one for each bank. Consistent with Figure 2, on average, banks allocate resources more to high-risk assets. The *Payoff* and *Default probability* of each type of risky asset and their *Default correlation* are the average macro credit information from our estimation. Consistent with the assumptions in the model, high-risk assets have higher probability of default and payoff compared to low-risk assets. These variables are used to test whether banks do absorb macro credit information in the expected direction.

*Basel II* is a dummy variable for the quarters since the agreed-upon text for Basel II was released;

**Table 1:** Summary statistics of the data

| VARIABLES | Observations | Mean | Std. Dev. | Min. | Max. |
|---|---|---|---|---|---|
| Proportion of high-risk assets | 59,441 | 0.598 | 0.163 | 0 | 1 |
| Proportion of low-risk assets | 59,441 | 0.402 | 0.163 | 0 | 1 |
| Payoff of low-risk assets | 59,441 | 0.036 | 0.014 | 0.011 | 0.058 |
| Payoff of high-risk assets | 59,441 | 0.060 | 0.022 | 0.025 | 0.112 |
| Default prob. of low-risk assets | 59,441 | 0.001 | 0.001 | 0 | 0.006 |
| Default prob. of high-risk assets | 59,441 | 0.019 | 0.015 | 0.001 | 0.048 |
| Default correlation | 59,441 | 0.008 | 0.011 | -0.005 | 0.058 |
| Size | 59,441 | 6.532 | 82.17 | 0.0004 | 2,573 |
| Ln(Size) | 59,441 | -0.686 | 1.491 | -7.927 | 7.853 |
| Equity ratio | 59,441 | 0.107 | 0.065 | 0.0002 | 1.183 |
| Nonperforming assets | 55,479 | 0.020 | 0.031 | 3.98E-07 | 0.476 |
| Cost-to-income ratio | 59,049 | 0.804 | 0.709 | -14.93 | 84.67 |
| Noninterest income | 59,007 | 0.155 | 0.123 | 3.20E-05 | 1 |
| Return on assets | 19,221 | 0.007 | 0.017 | -0.482 | 0.273 |
| Liquid assets | 59,436 | 0.218 | 0.127 | 0.0002 | 1 |

*Proportion of high-risk assets* and *Proportion of low-risk assets* are the proportions of banks' allocations within risky assets. Therefore, for each bank, they sum to one. *Payoff*, *Default prob.*, and *Default correlation* are based on the average macro credit information on the risky assets from our estimation. Other bank-level variables are *Size* (total assets in billions of dollars), *Equity ratio* (the ratio of total equity to total assets),, *Nonperforming assets* (the ratio of nonperforming assets to total assets), *Cost-to-income ratio*, *Noninterest income* (the ratio of the absolute value of noninterest income to the sums of the absolute values of noninterest and interest income), *Return on assets*, and *Liquid assets* (the ratio of liquid assets to total assets). The estimated payoffs, default probabilities, and default correlation (in percentage) are valued at the beginning of each quarter.

unless stated otherwise, we use the third quarter of 2004 as the starting date of Basel II. As a robustness check, we re-run all our estimations with the date of the first implementation, i.e., the second quarter of 2008 as the starting date of Basel II and all the results remain qualitatively the same. One important reason for using the third quarter of 2004 rather than the second quarter of 2008 as the starting date of Basel II in the main specification is the possible confounding effects of the financial crisis if we would have used the latter alternative. We use *Basel II* as a proxy for a tightened capital requirement. This approximation is applicable since an asset's risk is valued by the type of its obligors under Basel I instead of by the actual risk of the obligors. For example, assets involving banks in OECD countries are classified as 20% risk category under Basel I; however, among these, those whose obligors have high credit risk will fall to 50 % or 100 % risk category under Basel II. The remaining variables are bank-level controls.

Table 2 presents our baseline results. We find that, except for the payoff of low-risk assets, all key variables are significant and all have the signs that we expect from Proposition 1 in the appendix. Since the correlation is about 0.75, however, it is hard to distinguish between the effects of the returns on high-risk and low-risk assets, which might explain the insignificance of the coefficient related to the return on the low-risk assets. The positive effect of the Basel II dummy suggests that the stricter regulation under Basel II actually led to a higher proportion of high-risk assets within their risky portfolios, which seems counter-intuitive at first sight, but this is exactly what might happen according to Proposition 3: we show that it could both lead to lower and, more interestingly, higher proportions of high-risk assets, and it is in fact this latter effect that is present in our data.

As a further test of the effect of risk-taking of a stricter capital regulation, we estimate our model from the second specification in Table 2 for unconstrained banks in the Basel I period and use the predictions from that model in the Basel II period to investigate whether banks that become constrained in the Basel II period take higher or lower risks (according to Proposition 3, it could go either way). More specifically, we identify banks that become constrained under Basel II by using the following approach:

(i) We use observations on unconstrained banks in the Basel I period to estimate our model in specification 2 in Table 2.

(ii) We use the parameter estimates from (i) to predict how banks would behave in the Basel II period given Basel I-type reactions. If a bank should have been unconstrained in the Basel II period according to our prediction, but is actually constrained, we consider it as having become constrained as we move from Basel I to Basel II.

Using this method, we single out five banks that become constrained in the Basel II period. For this (admittedly limited) sample, we find that those five banks actually all hold higher shares of high-risk assets in their risky portfolios as compared to what they would be expected to do under Basel I and the difference between the actual and the expected allocation to high-risk assets is significantly positive (see Table 3). That is, the stricter regulation incurred by Basel II actually led all considered banks to increase the share of high-risk assets, something which might occur according to Proposition 3 in Appendix A. In the baseline setting, we include the financial crisis period, but we get a similar result if we exclude it – the only difference being that the number of banks that become constrained is reduced to three. The result is also robust to using the second quarter of 2008 as the starting date of Basel II.

To identify banks' portfolio inertia, and make a comparison between the Basel I and Basel II periods in this respect, we estimate a dynamic model with lagged decision variables as well as lagged macro variables (see Table 4). Because our data starts in the first quarter of 2002, we cannot use the third quarter of 2004 as the starting point of Basel II when estimating the dynamic model. Instead, we use our alternative date: the second quarter of 2008. Given that none of our previous results were sensitive to the assumed starting date of Basel II, we do not think that this choice will substantially affect the results.

As seen in Table 4, we find evidence of positive dynamic feedback effects: all significant lagged decision variables have positive coefficients. That is, our results suggest that, during both periods, banks tend to use previous portfolio decisions as benchmarks. Further, we find that there is a stronger dependence on lagged decision variables under Basel I whereas, under Basel II, there is a stronger dependence on lagged macro variables. One potential explanation for this latter result is Basel II's joint focus on risk-sensitive capital allocation and quantification of various types of risk (credit, operational and market risk) based on data and formal techniques, which result in a closer match to assets' actual risks compared to Basel I.

[Insert Table 2 about here]

[Insert Table 3 about here]

[Insert Table 4 about here]

## 5. Conclusion

This paper explicitly investigates the credit risk of banks' assets and addresses banks' portfolio allocations under risk-based capital regulation. Drawing on the credit portfolio optimization literature, we disentangle the effects of risk-based capital regulation on the credit risk of banks' assets.

When risk-based capital regulation is binding, the risk weightings assigned by the regulator affect the original measures of risk and valuation of assets: namely, volatility around expected loss due to default risk and Sharpe ratio, respectively. This raises concerns that, if the risk weightings are not consistent with the assets' true risk measures, there could be opportunities for regulatory arbitrage so that banks invest more in assets with a high level of true risk but a low regulatory risk weighting. If the regulator imposes a new, and more stringent, regulation the bank whose capital is already constrained will skew the risky portfolio to high-risk, high-earning assets, provided that the reward-to-regulatory-cost ratio of high-risk assets is higher than that of low-risk assets. If the reward-to-regulatory cost ratio of high-risk assets is instead lower than that of low-risk assets, we get the opposite result.

The empirical tests support our hypotheses. Due to business confidentiality, detailed data on each asset of each bank are not available. Yet, the average macro information on payoffs and credit risk of assets in each risk category that we estimate is very helpful in explaining banks' actual asset choices. More specifically, the tests support the predictions of a flight towards higher returns and an avoidance of default risk. We also find that banks reacted to the implementation of the stricter Basel II rules by holding a higher fraction of high-risk assets within their risky portfolios. Further, we show empirically that banks' decisions tend to be positively related to previous decisions, i.e., we find evidence of portfolio inertia.

Our study contributes to the literature and to ongoing debates on banks' risk taking and capital regulation from the perspective of credit risk, which, it is hoped, paves a way for future research on banks' asset risk. For example, our empirical analysis could be enriched by using detailed data on assets at the individual-bank level.

## Appendix A: A simple model of banks' asset allocation

Here, we present a simple, one-period model of banks' asset allocation, where we assume that the bank (embodying the behavior of its manager) maximizes the expected utility of its future cash profit, using a quadratic elementary utility function.

### *A.1. Model set-up*

The random cash flow, $\widetilde{CF}$, of a risky asset is $(1+C)(1-\widetilde{Z})$, where $C$ is the payoff of the asset if not defaulted and $\widetilde{Z}$ is a variable for a default event with Bernoulli distribution, which takes value 1 with probability $p$ if default happens[18] and 0 otherwise. That is, there are two credit states, default and no default, and

$$\widetilde{Z} = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p. \end{cases} \tag{A.1}$$

The third and fourth moments of the Bernoulli distribution are functions of its mean and variance. Therefore, we can use the first two moments to represent the distribution. Naturally, the measure of risk is the volatility around the expected loss (mean) and an approximation of unexpected loss, which is a usual term in the credit-portfolio literature.

The expected value and variance of its cash flow are

$$\begin{aligned} &E[\widetilde{CF}] = (1+C) - E[\widetilde{Z}](1+C) = (1-p)(1+C) \text{ and} \\ &\text{Var}[\widetilde{CF}] = \text{Var}[\widetilde{Z}](1+C)^2 = p(1-p)(1+C)^2, \text{ respectively,} \end{aligned} \tag{A.2}$$

where $E$ stands for *Expectation* and Var for *Variance*. To ensure that the expected utility is decreasing

[18]For simplification, the recovery rate is assumed to be zero.

as default probability increases, we only consider $p < 0.5$, which is consistent with the estimated probabilities of default (far less than 0.5) in the empirical examination (Section 4). Then, the loss due to default risk is captured by a positively skewed Bernoulli distribution.

The cash-flow covariance of the two types of risky assets in the model – high-risk, high-earning assets with type h (*high*) and low-risk, low-earning assets with type l (*low*) – is

$$\mathrm{Cov}[\widetilde{Z}_{\mathrm{h}}, \widetilde{Z}_{\mathrm{l}}] = p_B - p_{\mathrm{h}} p_{\mathrm{l}} = \rho \sqrt{p_{\mathrm{h}}(1-p_{\mathrm{h}}) p_{\mathrm{l}}(1-p_{\mathrm{l}})}, \tag{A.3}$$

where $p_B$ and $\rho$ are the pair-wise probability of default and the default correlation between the two types, respectively.

The bank's utility function, which embodies the manager's risk aversion, is given by

$$u(\widetilde{\pi}) = 2a\widetilde{\pi} - \widetilde{\pi}^2, \tag{A.4}$$

where $\widetilde{\pi}$ is the random cash profit at the end of the decision period:

$$\widetilde{\pi} = A_{\mathrm{h}} \widetilde{CF}_{\mathrm{h}} + A_{\mathrm{l}} \widetilde{CF}_{\mathrm{l}} + G(1 + r_{\mathrm{f}}) - D r_{\mathrm{D}}. \tag{A.5}$$

Here, $\widetilde{CF}_{\mathrm{h}}$ and $\widetilde{CF}_{\mathrm{l}}$ are the random cash flows of high-risk, high-earning and low-risk, low-earning assets, respectively; $A_{\mathrm{h}}$ and $A_{\mathrm{l}}$ are their respective amounts in dollars; $G$ is the amount of the risk-free asset with return $r_{\mathrm{f}}$; and $D$ is for deposits with rate $r_{\mathrm{D}}$.

Consequently, the decision problem for the bank manager is

$$\arg\max_{A_{\mathrm{h}}, A_{\mathrm{l}}, G} \{E[u(\widetilde{\pi})]\} = \arg\max_{A_{\mathrm{h}}, A_{\mathrm{l}}, G} \{2aE[\widetilde{\pi}] - (E[\widetilde{\pi}]^2 + \mathrm{Var}[\widetilde{\pi}])\}, \tag{A.6}$$

subject to

$$A_{\mathrm{h}} + A_{\mathrm{l}} + G = D + K \tag{A.7a}$$

$$A_{\mathrm{h}} \geqslant 0,\ A_{\mathrm{l}} \geqslant 0,\ G \geqslant 0 \tag{A.7b}$$

$$\frac{K}{W_{\mathrm{h}}A_{\mathrm{h}} + W_{\mathrm{l}}A_{\mathrm{l}}} \geqslant k \ \text{ and } \ 1 \geqslant W_{\mathrm{h}} > W_{\mathrm{l}} > 0. \tag{A.7c}$$

The objective function can be viewed as a function of $a$, $A_{\mathrm{h}}$, $A_{\mathrm{l}}$, $G$, $C_{\mathrm{h}}$, $C_{\mathrm{l}}$, $p_{\mathrm{h}}$, $p_{\mathrm{l}}$, $\rho$, $r_{\mathrm{f}}, D$, and $r_{\mathrm{D}}$. $a$ is positive, and $a \geq (1+C_{\mathrm{h}})(D+K) - Dr_{\mathrm{D}} > 0$, which ensures a positive marginal utility of cash profit. $C_{\mathrm{h}}$ and $C_{\mathrm{l}}$ are payoffs of asset types h and l, respectively – $1 \geq C_{\mathrm{h}} > C_{\mathrm{l}} > r_{\mathrm{f}} > 0$ – and $p_{\mathrm{h}}$ and $p_{\mathrm{l}}$ are their respective probabilities of default – $0.5 > p_{\mathrm{h}} > p_{\mathrm{l}} > 0$. $K$ stands for capital and the first restriction (Equation (A.7a)) states the balance-sheet constraint. $W_{\mathrm{h}}$ and $W_{\mathrm{l}}$ are the risk weightings for risky assets used in the calculation of the total risk-weighted assets: They are constant and determined by the regulator, and, by definition, $1 \geq W_{\mathrm{h}} > W_{\mathrm{l}} > 0$. $k$ denotes the minimum risk-based capital ratio determined by the regulator and the third restriction (Equation (A.7c)) expresses the regulatory capital constraint. In practice, actual defaults are positively, but not perfectly positively, correlated (Kealhofer and Bohn, 2001). Hence, default correlation, $\rho$ here, belongs to interval $(0,1)$.

In addition, the following subsections are based on solutions to the above maximization problem when the risky assets generate positive excess cash flows over risk-free assets: That is, $X_{\mathrm{h}} \equiv (1-p_{\mathrm{h}})(1+C_{\mathrm{h}}) - (1+r_{\mathrm{f}}) > X_{\mathrm{l}} \equiv (1-p_{\mathrm{l}})(1+C_{\mathrm{l}}) - (1+r_{\mathrm{f}}) > 0$.

*A.2. Optimal portfolio allocation when the capital requirement is not binding*

When the capital requirement is not binding, we get the following inner solution to the maximization problem (Equation (A.6)).

$$A_{\mathrm{h}}^{*} = \frac{(a-B)(SR_{\mathrm{h}} - \rho SR_{\mathrm{l}})}{(SR_{\mathrm{h}}^{2} + SR_{\mathrm{l}}^{2} - 2\rho SR_{\mathrm{h}}SR_{\mathrm{l}} + 1 - \rho^{2})\sqrt{V_{\mathrm{h}}}} \tag{A.8}$$

$$A_{\mathrm{l}}^* = \frac{(a-B)(SR_{\mathrm{l}} - \rho SR_{\mathrm{h}})}{(SR_{\mathrm{h}}^2 + SR_{\mathrm{l}}^2 - 2\rho SR_{\mathrm{h}} SR_{\mathrm{l}} + 1 - \rho^2)\sqrt{V_{\mathrm{l}}}} \tag{A.9}$$

$$G^* = D + K - \frac{(a-B)[SR_{\mathrm{h}}(\sqrt{V_{\mathrm{l}}} - \rho\sqrt{V_{\mathrm{h}}}) + SR_{\mathrm{l}}(\sqrt{V_{\mathrm{h}}} - \rho\sqrt{V_{\mathrm{l}}})]}{(SR_{\mathrm{h}}^2 + SR_{\mathrm{l}}^2 - 2\rho SR_{\mathrm{h}} SR_{\mathrm{l}} + 1 - \rho^2)\sqrt{V_{\mathrm{h}} V_{\mathrm{l}}}}, \tag{A.10}$$

where $B \equiv (D+K)(1+r_{\mathrm{f}}) - Dr_{\mathrm{D}} < a$,[19] $SR_{\mathrm{h}} > \rho SR_{\mathrm{l}}$, $SR_{\mathrm{l}} > \rho SR_{\mathrm{h}}$ and $D+K$ is sufficiently large, so that all three quantities above are positive. $V$ stands for cash-flow variance, and $SR$ for Sharpe ratio,[20] which is the ratio of excess cash flow over the risk-free asset ($X$) to cash-flow volatility ($\sqrt{V}$).

The optimal allocations to the risky assets are determined by their Sharpe ratios, cash-flow variances, default correlation, and the bank's risk aversion. Finally, the balance-sheet constraint controls the investment in risk-free assets.

From now on, we consider a change in the risk or payoff of one type of the risky assets or in default correlation, and derive how the optimal allocation adjusts. This is to reveal a more dynamic picture of how the bank restructures the portfolio of different assets when the conditional information on assets changes. Here, we use the two-fund separation theorem (Markowitz, 1952). The two funds refer to the risky fund, which is comprised of high-risk, high-earning and low-risk, low-earning assets, and the risk-free fund, comprised of essentially risk-free assets. Then, the risky fund is the tangential portfolio on the capital market line, which is the ray from the risk-free cash flow with a tangency to the mean-variance efficient frontier of risky assets. Within the risky fund, the portfolio weights of type $h$ and type $l$ assets are defined as $\omega_{\mathrm{h}}^* = \frac{A_{\mathrm{h}}^*}{A_{\mathrm{h}}^* + A_{\mathrm{l}}^*}$ and $\omega_{\mathrm{l}}^* = \frac{A_{\mathrm{l}}^*}{A_{\mathrm{h}}^* + A_{\mathrm{l}}^*}$, respectively. For the portfolio composed of the risky and risk-free fund, the amounts of the allocations $A_{\mathrm{h}}^* + A_{\mathrm{l}}^*$ and $G^*$ represent their relative portfolio weights, since the bank's size does not change.

[19]Note that, because we assume $a \geq (1 + C_{\mathrm{h}})(D+K) - Dr_{\mathrm{D}} > 0$, $B < a$.

[20]This reward-to-variability ratio (Sharpe, 1966) is modified according to the settings in our model.

*The risky fund*

The following proposition is derived from the first derivative of the optimal weight of the high-risk, high-earning asset ($\omega_h^*$) with respect to the payoff or default probability of any risky asset, or default correlation. Obviously, the weight of the low-risk, low-earning asset ($\omega_l^*$) would consequently change in the opposite direction.

**Proposition 1.** *Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if*

(a) *its payoff, $C_h$, increases (decreases); or*

(b) *its probability of default, $p_h$, decreases (increases); or*

(c) *the payoff of the low-risk asset, $C_l$, decreases (increases); or*

(d) *the default probability of the low-risk asset, $p_l$, increases (decreases); or*

(e) i. *the default correlation, $\rho$, increases (decreases) provided that $SR_h > SR_l$; or*

ii. *the default correlation, $\rho$, decreases (increases) provided that $SR_h < SR_l$.*

*Proof:*

*(a) Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if its payoff, $C_h$, increases (decreases).*

$$
\begin{aligned}
\frac{\partial \omega_{\mathrm{h}}^{*}}{\partial C_{\mathrm{h}}} &= \frac{\partial \dfrac{A_{\mathrm{h}}^{*}}{A_{\mathrm{h}}^{*}+A_{\mathrm{l}}^{*}}}{\partial C_{\mathrm{h}}} \\
&= \frac{(A_{\mathrm{h}}^{*})^{2}\sqrt{p_{\mathrm{h}}(1-p_{\mathrm{h}})}}{(A_{\mathrm{h}}^{*}+A_{\mathrm{l}}^{*})^{2}(E_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}-\rho E_{\mathrm{l}}\sqrt{V_{\mathrm{h}}})^{2}\sqrt{V_{\mathrm{l}}}}\{\rho(E_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}-E_{\mathrm{h}}\sqrt{V_{\mathrm{l}}})^{2} \\
&\quad + (1-\rho)E_{\mathrm{l}}\sqrt{V_{\mathrm{h}}V_{\mathrm{l}}}[\rho(1-p_{\mathrm{h}})(1+C_{\mathrm{h}})+2(1+r_{\mathrm{f}})-(1-p_{\mathrm{h}})(1+C_{\mathrm{h}})]\} \\
&> 0
\end{aligned} \tag{A.11}
$$

since $1>\rho>0$ and $2(1+r_{\mathrm{f}})\geq 2>(1-p_{\mathrm{h}})(1+C_{\mathrm{h}})$. Then $\frac{\partial \omega_{\mathrm{h}}^{*}}{\partial C_{\mathrm{h}}}$ is positive. That is, the bank invests proportionally more (less) in high-risk assets, *ceteris paribus*, if its yield $C_{\mathrm{h}}$ increases (decreases).

*(b) Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if its probability of default,* $p_{\mathrm{h}}$*, decreases (increases).*

$$
\begin{aligned}
\frac{\partial \omega_{\mathrm{h}}^{*}}{\partial p_{\mathrm{h}}} &= \frac{\partial \dfrac{A_{\mathrm{h}}^{*}}{A_{\mathrm{h}}^{*}+A_{\mathrm{l}}^{*}}}{\partial p_{\mathrm{h}}} \\
&= -\frac{(A_{\mathrm{h}}^{*})^{2}}{(A_{\mathrm{h}}^{*}+A_{\mathrm{l}}^{*})^{2}(E_{\mathrm{h}}V_{\mathrm{l}}-\rho E_{\mathrm{l}}\sqrt{V_{\mathrm{h}}V_{\mathrm{l}}})^{2}}\{(1-\rho^{2})E_{\mathrm{l}}V_{\mathrm{h}}V_{\mathrm{l}}(1+C_{\mathrm{h}}) \\
&\quad + \frac{V_{\mathrm{l}}}{2}(1-2p_{\mathrm{h}})(1+C_{\mathrm{h}})^{2}[E_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}(SR_{\mathrm{h}}-\rho SR_{\mathrm{l}})+E_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}(SR_{\mathrm{l}}-\rho SR_{\mathrm{h}})]\} \\
&< 0
\end{aligned} \tag{A.12}
$$

since $p_{\mathrm{h}}<0.5$, $SR_{\mathrm{h}}-\rho SR_{\mathrm{l}}>0$ and $SR_{\mathrm{l}}-\rho SR_{\mathrm{h}}>0$ (as $A_{\mathrm{h}}^{*}>0$ and $A_{\mathrm{l}}^{*}>0$). Hence, $\frac{\partial \omega_{\mathrm{h}}^{*}}{\partial p_{\mathrm{h}}}<0$. That is, the bank invests proportionally more (less) in high-risk assets, *ceteris paribus*, if its probability of default, $p_{\mathrm{h}}$, decreases (increases).

*(c) Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if the payoff of low-risk asset,* $C_{\mathrm{l}}$*, decreases (increases).*

It can be shown as in the proof for statement *(a)*.

*(d) Within the risky fund, the bank invests proportionally more (less) in high-risk asset, ceteris paribus, if the default probability of low-risk asset, $p_{\mathrm{l}}$, increases (decreases).*

It can be shown as in the proof for statement *(b)*.

*(e) Within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if i. the default correlation, $\rho$, increases (decreases) provided that $SR_{\mathrm{h}} > SR_{\mathrm{l}}$; or ii. the default correlation, $\rho$, decreases (increases) provided that $SR_{\mathrm{h}} < SR_{\mathrm{l}}$.*

$$\frac{\partial \omega_{\mathrm{h}}^*}{\partial \rho} = \frac{\partial \dfrac{A_{\mathrm{h}}^*}{A_{\mathrm{h}}^* + A_{\mathrm{l}}^*}}{\partial \rho} = \frac{(A_{\mathrm{h}}^*)^2 V_{\mathrm{h}} \sqrt{V_{\mathrm{h}} V_{\mathrm{l}}}}{(A_{\mathrm{h}}^* + A_{\mathrm{l}}^*)^2 (E_{\mathrm{h}} \sqrt{V_{\mathrm{l}}} - \rho E_{\mathrm{l}} \sqrt{V_{\mathrm{h}}})^2} (SR_{\mathrm{h}}^2 - SR_{\mathrm{l}}^2) > 0 \tag{A.13}$$

if $SR_{\mathrm{h}} > SR_{\mathrm{l}}$. Then the bank invests proportionally more (less) in high-risk assets, *ceteris paribus*, if the default correlation, $\rho$, increases (decreases), given that $SR_{\mathrm{h}} > SR_{\mathrm{l}}$. The sign of the above expression is reversed if instead $SR_{\mathrm{h}} < SR_{\mathrm{l}}$. □

As expected, the bank invests more in high-risk, high-earning assets when the asset generates higher payoff or its obligor has a lower probability of defaulting, or the other risky asset generates a lower payoff or its obligor has a higher probability of defaulting, *ceteris paribus*. In short, a high-risk, high-earning asset acts as a substitute for a low-risk, low-yield asset, and provides a natural hedge against losses stemming from low-risk assets.

When default correlation increases, the bank allocates more to high-risk, high-earning assets if their Sharpe ratio is larger than that of low-risk, low-yield assets. That is, when the two types of assets are more likely to default at the same time, the best strategy is to compare their Sharpe ratios and choose the asset type with a higher Sharpe ratio.

*The risk-free fund*

For the risky fund as a single asset, there is no measure of its overall payoff or probability of default. Therefore, as in Section A.2, we consider any change in the payoff or default probability of any risky asset or in the default correlation, which measures how the earning or risk of the whole fund varies. The following proposition is derived from the first derivative of the optimal investment in a risk-free asset ($G^*$ (Equation (A.10)) with respect to each of these measures. Obviously, the allocation to the risky fund ($A_{\mathrm{h}}^* + A_{\mathrm{l}}^*$) would consequently change in the opposite direction. Recall that the amounts of the allocations in different funds represent their relative portfolio weights since the bank's size does not change.

**Proposition 2.** *The bank invests more (less) in the risk-free fund, ceteris paribus, if*

(a) *the risk-free rate $r_{\mathrm{f}}$ increases (decreases); or*

(b) *the payoff of the high-risk asset, $C_{\mathrm{h}}$, increases (decreases), given that $\rho\sqrt{V_{\mathrm{h}}} \geq \sqrt{V_{\mathrm{l}}}$; or*

(c) *the default probability of the high-risk asset, $p_{\mathrm{h}}$, decreases (increases), given that $\dfrac{\rho\sqrt{V_{\mathrm{h}}}}{\sqrt{V_{\mathrm{l}}}} \geq \dfrac{2X_{\mathrm{h}}}{X_{\mathrm{h}} + X_{\mathrm{l}}} > 1$; or*

(d) *the payoff of the low-risk asset, $C_{\mathrm{l}}$, decreases (increases), given that $\dfrac{\rho\sqrt{V_{\mathrm{h}}}}{\sqrt{V_{\mathrm{l}}}} \geq \dfrac{X_{\mathrm{h}} + 1 + r_{\mathrm{f}}}{X_{\mathrm{l}} + 1 + r_{\mathrm{f}}}$, $\rho^2 X_{\mathrm{h}} \geq X_{\mathrm{l}}$, $\rho(1 + r_{\mathrm{f}}) > X_{\mathrm{l}}$ and $\dfrac{X_{\mathrm{h}}^2}{X_{\mathrm{l}}^2} \geq \dfrac{V_{\mathrm{h}} - \rho\sqrt{V_{\mathrm{h}} V_{\mathrm{l}}}}{V_{\mathrm{l}} - \rho^2 V_{\mathrm{l}}}$; or*

(e) *the default probability of the low-risk asset, $p_{\mathrm{l}}$, increases (decreases), given that $\rho\sqrt{V_{\mathrm{h}}} \geq \sqrt{V_{\mathrm{l}}}$ and $SR_{\mathrm{l}}^2 \leq 1 - \rho^2$; or*

(f) *the default correlation, $\rho$, increases (decreases), assuming $\rho\sqrt{V_{\mathrm{h}}} \leq \sqrt{V_{\mathrm{l}}}$ and $SR_{\mathrm{h}} \geq SR_{\mathrm{l}}$.*

*Proof:*

*(a) The bank invests more (less) in the risk-free fund, ceteris paribus, if the risk-free rate, $r_{\mathrm{f}}$, increases (decreases).*

$$\frac{\partial G^*}{\partial r_{\mathrm{f}}} = (D+K)\frac{\sqrt{V_{\mathrm{l}}}(X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} - \rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}) + \sqrt{V_{\mathrm{h}}}(X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}} - \rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}})}{M} > 0 \tag{A.14}$$

since $X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} > \rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}$ and $X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}} > \rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}$ (as $A_{\mathrm{h}}^* > 0$ and $A_{\mathrm{l}}^* > 0$), where $M \equiv X_{\mathrm{h}}^2 V_{\mathrm{l}} + X_{\mathrm{l}}^2 V_{\mathrm{h}} - 2\rho X_{\mathrm{h}} X_{\mathrm{l}}\sqrt{V_{\mathrm{h}} V_{\mathrm{l}}} + (1-\rho^2)V_{\mathrm{h}}V_{\mathrm{l}} = X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}(X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} - \rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}) + X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}(X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}} - \rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}) + (1-\rho^2)V_{\mathrm{h}}V_{\mathrm{l}} > 0$.

*(b) The bank invests more (less) in the risk-free fund, ceteris paribus, if the payoff of the high-risk asset, $C_{\mathrm{h}}$, increases (decreases), given that $\rho\sqrt{V_{\mathrm{h}}} \geq \sqrt{V_{\mathrm{l}}}$.*

$$\begin{aligned}\frac{\partial G^*}{\partial C_{\mathrm{h}}} &= \frac{(a-B)\sqrt{V_{\mathrm{l}}}(X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} - \rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}})}{M(1+C_{\mathrm{h}})} \\ &+ \frac{(a-B)\sqrt{V_{\mathrm{l}}}(1+r_{\mathrm{f}})(X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} - \rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}})\{X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}(\sqrt{V_{\mathrm{h}}} - \rho\sqrt{V_{\mathrm{l}}}) + \sqrt{V_{\mathrm{l}}}(X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} - \rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}})\}}{M^2(1+C_{\mathrm{h}})} \\ &+ \frac{(a-B)\sqrt{V_{\mathrm{l}}}(1+r_{\mathrm{f}})\{\sqrt{V_{\mathrm{h}}V_{\mathrm{l}}}(X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}} - \rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}})(X_{\mathrm{h}} - X_{\mathrm{l}}) + (1-\rho^2)V_{\mathrm{h}}V_{\mathrm{l}}(\rho\sqrt{V_{\mathrm{h}}} - \sqrt{V_{\mathrm{l}}})\}}{M^2(1+C_{\mathrm{h}})}\end{aligned} \tag{A.15}$$

Since $a > B$, $X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} > \rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}$, $\sqrt{V_{\mathrm{h}}} > \rho\sqrt{V_{\mathrm{l}}}$ (as $V_{\mathrm{h}} > V_{\mathrm{l}}$ and $\rho < 1$), $X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}} > \rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}$, and $X_{\mathrm{h}} > X_{\mathrm{l}}$, $\dfrac{\partial G^*}{\partial C_{\mathrm{h}}} > 0$, given that $\rho\sqrt{V_{\mathrm{h}}} \geq \sqrt{V_{\mathrm{l}}}$.

*(c) The bank invests more (less) in the risk-free fund, ceteris paribus, if the default probability of high-risk asset, $p_{\mathrm{h}}$, decreases (increases), given that $\dfrac{\rho\sqrt{V_{\mathrm{h}}}}{\sqrt{V_{\mathrm{l}}}} \geq \dfrac{2X_{\mathrm{h}}}{X_{\mathrm{h}} + X_{\mathrm{l}}}$.*

$$
\begin{aligned}
\frac{\partial G^*}{\partial p_\mathrm{h}} &= \frac{-(a-B)(1+C_\mathrm{h})\sqrt{V_\mathrm{l}}}{M^2} \\
&\times \Big\{ \sqrt{V_\mathrm{h}V_\mathrm{l}}(X_\mathrm{l}\sqrt{V_\mathrm{h}} - \rho X_\mathrm{h}\sqrt{V_\mathrm{l}})(X_\mathrm{h}-X_\mathrm{l}) + (X_\mathrm{h}\sqrt{V_\mathrm{l}} - \rho X_\mathrm{l}\sqrt{V_\mathrm{h}})^2\sqrt{V_\mathrm{l}} \\
&+ X_\mathrm{l}\sqrt{V_\mathrm{h}}(X_\mathrm{h}\sqrt{V_\mathrm{l}} - \rho X_\mathrm{l}\sqrt{V_\mathrm{h}})(\sqrt{V_\mathrm{h}} - \rho\sqrt{V_\mathrm{l}}) + (1-\rho^2)V_\mathrm{h}V_\mathrm{l}(\rho\sqrt{V_\mathrm{h}} - \sqrt{V_\mathrm{l}}) \Big\} \\
&- \frac{(a-B)(1-2p_\mathrm{h})\sqrt{V_\mathrm{l}}}{2\sqrt{V_\mathrm{h}}M^2} \\
&\times \Big\{ (X_\mathrm{h}-X_\mathrm{l}) \Big[ X_\mathrm{l}\sqrt{V_\mathrm{h}}(X_\mathrm{h}\sqrt{V_\mathrm{l}} - \rho X_\mathrm{l}\sqrt{V_\mathrm{h}}) + X_\mathrm{h}\sqrt{V_\mathrm{l}}(X_\mathrm{l}\sqrt{V_\mathrm{h}} - \rho X_\mathrm{h}\sqrt{V_\mathrm{l}}) \Big] \\
&+ \sqrt{V_\mathrm{h}}V_\mathrm{l}(1-\rho^2)\Big[ (X_\mathrm{h}+X_\mathrm{l})\rho\sqrt{V_\mathrm{h}} - 2X_\mathrm{h}\sqrt{V_\mathrm{l}} \Big] \Big\}.
\end{aligned}
\tag{A.16}
$$

The term within the first pair of large braces is negative given $\rho\sqrt{V_\mathrm{h}} \geq \sqrt{V_\mathrm{l}}$ since $a > B$, $X_\mathrm{l}\sqrt{V_\mathrm{h}} > \rho X_\mathrm{h}\sqrt{V_\mathrm{l}}$, $X_\mathrm{h} > X_\mathrm{l}$, $X_\mathrm{h}\sqrt{V_\mathrm{l}} > \rho X_\mathrm{l}\sqrt{V_\mathrm{h}}$, and $\sqrt{V_\mathrm{h}} > \rho\sqrt{V_\mathrm{l}}$. The term within the second pair of large braces is negative given that $\frac{\rho\sqrt{V_\mathrm{h}}}{\sqrt{V_\mathrm{l}}} \geq \frac{2X_\mathrm{h}}{X_\mathrm{h}+X_\mathrm{l}}$, since $p_\mathrm{h} < 0.5$. Therefore, $\frac{\partial G^*}{\partial p_\mathrm{h}} < 0$ given that $\frac{\rho\sqrt{V_\mathrm{h}}}{\sqrt{V_\mathrm{l}}} \geq \frac{2X_\mathrm{h}}{X_\mathrm{h}+X_\mathrm{l}}$.

*(d) The bank invests more (less) in the risk-free fund, ceteris paribus, if the payoff of the low-risk asset, $C_\mathrm{l}$, decreases (increases), given that $\frac{\rho\sqrt{V_\mathrm{h}}}{\sqrt{V_\mathrm{l}}} \geq \frac{X_\mathrm{h}+1+r_\mathrm{f}}{X_\mathrm{l}+1+r_\mathrm{f}}$, $\rho^2 X_\mathrm{h} \geq X_\mathrm{l}$, $\rho(1+r_\mathrm{f}) > X_\mathrm{l}$ and $\frac{X_\mathrm{h}^2}{X_\mathrm{l}^2} \geq \frac{V_\mathrm{h} - \rho\sqrt{V_\mathrm{h}V_\mathrm{l}}}{V_\mathrm{l} - \rho^2 V_\mathrm{l}}$.*

$$
\begin{aligned}
\frac{\partial G^*}{\partial C_\mathrm{l}} &= -\frac{a-B}{M^2(1+C_\mathrm{l})} \Big\{ \rho X_\mathrm{h}^2\sqrt{V_\mathrm{h}}V_\mathrm{l}(X_\mathrm{h}\sqrt{V_\mathrm{l}} - \rho X_\mathrm{l}\sqrt{V_\mathrm{h}}) \\
&+ (1-\rho^2)V_\mathrm{h}V_\mathrm{l}\sqrt{V_\mathrm{h}}\Big[ \sqrt{V_\mathrm{h}}(1+r_\mathrm{f}-X_\mathrm{l}) - \rho\sqrt{V_\mathrm{l}}(1+r_\mathrm{f}-X_\mathrm{h}) \Big] \\
&+ X_\mathrm{h}X_\mathrm{l}V_\mathrm{h}\sqrt{V_\mathrm{l}}\Big[ (1+r_\mathrm{f})(\rho\sqrt{V_\mathrm{h}} - \sqrt{V_\mathrm{l}}) - (X_\mathrm{h}\sqrt{V_\mathrm{l}} - \rho X_\mathrm{l}\sqrt{V_\mathrm{h}}) \Big] \\
&+ (1+r_\mathrm{f})V_\mathrm{h}\Big[ (1-\rho^2)X_\mathrm{h}^2 V_\mathrm{l} - X_\mathrm{l}^2\sqrt{V_\mathrm{h}}(\sqrt{V_\mathrm{h}} - \rho\sqrt{V_\mathrm{l}}) \Big] \\
&+ \sqrt{V_\mathrm{h}}(X_\mathrm{l}\sqrt{V_\mathrm{h}} - \rho X_\mathrm{h}\sqrt{V_\mathrm{l}}) \\
&\times \Big[ (1+r_\mathrm{f})X_\mathrm{h}\sqrt{V_\mathrm{l}}(\rho\sqrt{V_\mathrm{h}} - \sqrt{V_\mathrm{l}}) - X_\mathrm{l}\sqrt{V_\mathrm{h}}(X_\mathrm{l}\sqrt{V_\mathrm{h}} - \rho X_\mathrm{h}\sqrt{V_\mathrm{l}}) \Big] \Big\}.
\end{aligned}
\tag{A.17}
$$

Within the large braces, the term in the first row is positive since $X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} > \rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}$; the term in the second row is positive since $\sqrt{V_{\mathrm{h}}} > \rho\sqrt{V_{\mathrm{l}}}$ and $1+r_{\mathrm{f}} > X_{\mathrm{h}} > X_{\mathrm{l}}$; the term in the third row is nonnegative given that $\dfrac{\rho\sqrt{V_{\mathrm{h}}}}{\sqrt{V_{\mathrm{l}}}} \geq \dfrac{X_{\mathrm{h}}+1+r_{\mathrm{f}}}{X_{\mathrm{l}}+1+r_{\mathrm{f}}}$; the term in the fourth row is nonnegative given that $\dfrac{X_{\mathrm{h}}^2}{X_{\mathrm{l}}^2} \geq \dfrac{V_{\mathrm{h}}-\rho\sqrt{V_{\mathrm{h}}V_{\mathrm{l}}}}{V_{\mathrm{l}}-\rho^2 V_{\mathrm{l}}}$ since $\sqrt{V_{\mathrm{h}}} > \rho\sqrt{V_{\mathrm{l}}}$. In addition, $\dfrac{X_{\mathrm{h}}^2}{X_{\mathrm{l}}^2} \geq \dfrac{V_{\mathrm{h}}-\rho\sqrt{V_{\mathrm{h}}V_{\mathrm{l}}}}{V_{\mathrm{l}}-\rho^2 V_{\mathrm{l}}}$ implies that $X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} > X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}$ if $\rho\sqrt{V_{\mathrm{h}}} > \sqrt{V_{\mathrm{l}}}$. Then the term in the last row is positive if $(1+r_{\mathrm{f}})(\rho\sqrt{V_{\mathrm{h}}}-\sqrt{V_{\mathrm{l}}}) \geq X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}-\rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}$, which is true given that $\rho\sqrt{V_{\mathrm{h}}} > \sqrt{V_{\mathrm{l}}}$, $\rho(1+r_{\mathrm{f}}) > X_{\mathrm{l}}$ and $\rho^2 X_{\mathrm{h}} \geq X_{\mathrm{l}}$.

Therefore, $\dfrac{\partial G^*}{\partial C_{\mathrm{l}}} < 0$ given that $\dfrac{\rho\sqrt{V_{\mathrm{h}}}}{\sqrt{V_{\mathrm{l}}}} \geq \dfrac{X_{\mathrm{h}}+1+r_{\mathrm{f}}}{X_{\mathrm{l}}+1+r_{\mathrm{f}}}$, $\rho^2 X_{\mathrm{h}} \geq X_{\mathrm{l}}$, $\rho(1+r_{\mathrm{f}}) > X_{\mathrm{l}}$, and $\dfrac{X_{\mathrm{h}}^2}{X_{\mathrm{l}}^2} \geq \dfrac{V_{\mathrm{h}}-\rho\sqrt{V_{\mathrm{h}}V_{\mathrm{l}}}}{V_{\mathrm{l}}-\rho^2 V_{\mathrm{l}}}$, as $a > B$.

*(e) The bank invests more (less) in the risk-free fund, ceteris paribus, if the default probability of the low-risk asset, $p_{\mathrm{l}}$, increases(decreases), given that $\rho\sqrt{V_{\mathrm{h}}} \geq \sqrt{V_{\mathrm{l}}}$ and $SR_{\mathrm{l}}^2 \leq 1-\rho^2$.*

$$
\begin{aligned}
\frac{\partial G^*}{\partial p_{\mathrm{l}}} &= \frac{(a-B)(1+C_{\mathrm{l}})\sqrt{V_{\mathrm{h}}}}{M^2} \\
&\times \Bigg\{ X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}(\rho\sqrt{V_{\mathrm{h}}}-\sqrt{V_{\mathrm{l}}})(2X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}-\rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}) + (1-\rho^2)X_{\mathrm{h}}^2 V_{\mathrm{l}}\sqrt{V_{\mathrm{h}}} \\
&+ V_{\mathrm{h}}(\sqrt{V_{\mathrm{h}}}-\rho\sqrt{V_{\mathrm{l}}})\left[(1-\rho^2)V_{\mathrm{l}}-X_{\mathrm{l}}^2\right] \Bigg\} \\
&+ \frac{(a-B)(1-2p_{\mathrm{l}})\sqrt{V_{\mathrm{h}}}}{2\sqrt{V_{\mathrm{l}}}M^2} \\
&\times \Bigg\{ (X_{\mathrm{h}}-X_{\mathrm{l}})\Big[X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}(X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}-\rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}) + X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}(X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}-\rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}})\Big] \\
&+ \sqrt{V_{\mathrm{l}}}V_{\mathrm{h}}(1-\rho^2)\Big[\rho\sqrt{V_{\mathrm{l}}}(X_{\mathrm{h}}-X_{\mathrm{l}}) + 2(X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}-\rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}})\Big] \Bigg\}.
\end{aligned}
\tag{A.18}
$$

Within the first pair of large braces, the term in the first row is positive given $\rho\sqrt{V_{\mathrm{h}}} \geq \sqrt{V_{\mathrm{l}}}$, since $X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}} > \rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}$; the term in the second row is positive given that $X_{\mathrm{l}}^2 \leq (1-\rho^2)V_{\mathrm{l}}$ – i.e., $SR_{\mathrm{l}}^2 \leq 1-\rho^2$ – since $\sqrt{V_{\mathrm{h}}} > \rho\sqrt{V_{\mathrm{l}}}$. Within the second pair of large braces, the term in the first row is positive since $X_{\mathrm{h}} > X_{\mathrm{l}}$, $X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}} > \rho X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}}$, and $X_{\mathrm{l}}\sqrt{V_{\mathrm{h}}} > \rho X_{\mathrm{h}}\sqrt{V_{\mathrm{l}}}$, which also implies that the term in the second row is positive.

Therefore, $\dfrac{\partial G^*}{\partial p_\mathrm{l}} > 0$ given that $\rho\sqrt{V_\mathrm{h}} \geq \sqrt{V_\mathrm{l}}$ and $SR_\mathrm{l}^2 \leq 1-\rho^2$, since $a > B$ and $p_\mathrm{l} < 0.5$.

*(f) The bank invests more (less ) in the risk-free fund, ceteris paribus, if the default correlation, $\rho$, increases (decreases), given that $\rho\sqrt{V_\mathrm{h}} \leq \sqrt{V_\mathrm{l}}$ and $SR_\mathrm{h} \geq SR_\mathrm{l}$.*

$$
\begin{aligned}
\frac{\partial G^*}{\partial \rho} &= \frac{(a-B)\sqrt{V_\mathrm{h}V_\mathrm{l}}}{M^2} \\
&\times \Big\{ \sqrt{V_\mathrm{h}V_\mathrm{l}}\Big[(\sqrt{V_\mathrm{h}}-\rho\sqrt{V_\mathrm{l}})(X_\mathrm{h}\sqrt{V_\mathrm{l}}-\rho X_\mathrm{l}\sqrt{V_\mathrm{h}})+(\sqrt{V_\mathrm{l}}-\rho\sqrt{V_\mathrm{h}})(X_\mathrm{l}\sqrt{V_\mathrm{h}}-\rho X_\mathrm{h}\sqrt{V_\mathrm{l}})\Big] \\
&+(X_\mathrm{h}-X_\mathrm{l})(X_\mathrm{h}^2V_\mathrm{l}-X_\mathrm{l}^2V_\mathrm{h})\Big\}
\end{aligned} \tag{A.19}
$$

Within the large braces, the term in the first row is positive given that $\rho\sqrt{V_\mathrm{h}} \leq \sqrt{V_\mathrm{l}}$, since $\sqrt{V_\mathrm{h}} > \rho\sqrt{V_\mathrm{l}}$, $X_\mathrm{h}\sqrt{V_\mathrm{l}} > \rho X_\mathrm{l}\sqrt{V_\mathrm{h}}$, and $X_\mathrm{l}\sqrt{V_\mathrm{h}} > \rho X_\mathrm{h}\sqrt{V_\mathrm{l}}$; the term in the second row is nonnegative given that $X_\mathrm{h}^2V_\mathrm{l} \geq X_\mathrm{l}^2V_\mathrm{h}$: i.e., $SR_\mathrm{h} \geq SR_\mathrm{l}$, since $X_\mathrm{h} > X_\mathrm{l}$. Therefore, $\dfrac{\partial G^*}{\partial \rho} > 0$ given that $\rho\sqrt{V_\mathrm{h}} \leq \sqrt{V_\mathrm{l}}$ and $SR_\mathrm{h} \geq SR_\mathrm{l}$. $\square$

This proposition shows that when a high-risk asset is far riskier than a low-risk asset in terms of cash-flow variance – i.e., $\rho\sqrt{V_\mathrm{h}} > \sqrt{V_\mathrm{l}}$ – the risk-free fund is a complement to the high-risk asset (in statements (b) and (c)) and a substitute for the low-risk asset (in statements (d) and (e)), given possible additional conditions: For instance, where the relative difference in variance is greater than that in excess cash flow, and some measures of excess cash flows are limited.

When a high-risk asset generates a lower payoff or is more likely to default, the bank invests less in high-risk assets and more in low-risk assets, as Proposition 1 tells us. However, when a high-risk asset is far riskier than a low-risk asset in terms of cash-flow variance, the increase of the investment in the low-risk asset is much more than the decrease of that in the high-risk asset, so that the investment in the risk-free asset actually decreases. Yet, a risk-free asset is always a substitute for a low-risk asset, since the low-risk asset is always less risky than a high-risk asset.

When default correlation increases, the bank invests more in the risk-free fund, given that the cash-flow variances of the risky assets are relatively close and a high-risk asset earns a higher Sharpe ratio than a low-risk asset. Here, the risk-free fund mitigates the risk that arises when both types of risky assets default at the same time.

### *A.3. Impact of risk-based capital regulation*

This section disentangles the effects of risk-based capital regulation on the bank's asset risk by analyzing how the bank changes its asset allocation when the regulator imposes a new and more stringent capital requirement in the situation where the bank's capital is already constrained by the current regulation or will become constrained by the new regulation.

*Optimal portfolio allocation when the capital constraint is currently binding*

For the bank that achieves the minimum capital requirement and whose capital is constrained, allocations to risky assets are restricted by the risk-based capital ratio, which is the ratio of capital to the total risk-weighted assets. Under this condition, we derive the following inner solution to the maximization problem (Equation (A.6)).

$$A^*_{\mathrm{bh}} = \frac{(a-B)\varphi_{\mathrm{l}}(\varphi_{\mathrm{l}}SR_{\mathrm{h}} - \varphi_{\mathrm{h}}SR_{\mathrm{l}}) - \dfrac{K}{k}\{SR_{\mathrm{l}}(\varphi_{\mathrm{l}}SR_{\mathrm{h}} - \varphi_{\mathrm{h}}SR_{\mathrm{l}}) + (\rho\varphi_{\mathrm{l}} - \varphi_{\mathrm{h}})\}}{(\varphi_{\mathrm{h}}^2 - 2\rho\varphi_{\mathrm{h}}\varphi_{\mathrm{l}} + \varphi_{\mathrm{l}}^2)\sqrt{V_{\mathrm{h}}} + (\varphi_{\mathrm{l}}SR_{\mathrm{h}} - \varphi_{\mathrm{h}}SR_{\mathrm{l}})^2\sqrt{V_{\mathrm{h}}}} \tag{A.20}$$

$$A^*_{\mathrm{bl}} = \frac{(a-B)\varphi_{\mathrm{h}}(\varphi_{\mathrm{h}}SR_{\mathrm{l}} - \varphi_{\mathrm{l}}SR_{\mathrm{h}}) - \dfrac{K}{k}\{SR_{\mathrm{h}}(\varphi_{\mathrm{h}}SR_{\mathrm{l}} - \varphi_{\mathrm{l}}SR_{\mathrm{h}}) + (\rho\varphi_{\mathrm{h}} - \varphi_{\mathrm{l}})\}}{(\varphi_{\mathrm{h}}^2 - 2\rho\varphi_{\mathrm{h}}\varphi_{\mathrm{l}} + \varphi_{\mathrm{l}}^2)\sqrt{V_{\mathrm{l}}} + (\varphi_{\mathrm{l}}SR_{\mathrm{h}} - \varphi_{\mathrm{h}}SR_{\mathrm{l}})^2\sqrt{V_{\mathrm{l}}}} \tag{A.21}$$

$$
\begin{aligned}
G_{\mathrm{b}}^{*} = & D + K - \frac{K}{kW_{\mathrm{l}}} + \frac{W_{\mathrm{h}} - W_{\mathrm{l}}}{W_{\mathrm{l}}} \\
& \times \frac{(a-B)\varphi_{\mathrm{l}}(\varphi_{\mathrm{l}} SR_{\mathrm{h}} - \varphi_{\mathrm{h}} SR_{\mathrm{l}}) - \frac{K}{k}\{SR_{\mathrm{l}}(\varphi_{\mathrm{l}} SR_{\mathrm{h}} - \varphi_{\mathrm{h}} SR_{\mathrm{l}}) + (\rho\varphi_{\mathrm{l}} - \varphi_{\mathrm{h}})\}}{(\varphi_{\mathrm{h}}^2 - 2\rho\varphi_{\mathrm{h}}\varphi_{\mathrm{l}} + \varphi_{\mathrm{l}}^2)\sqrt{V_{\mathrm{h}}} + (\varphi_{\mathrm{l}} SR_{\mathrm{h}} - \varphi_{\mathrm{h}} SR_{\mathrm{l}})^2\sqrt{V_{\mathrm{h}}}},
\end{aligned}
\tag{A.22}
$$

where b stands for *binding*, $W_{\mathrm{h}}$ and $W_{\mathrm{l}}$ are the risk weightings for risky assets used in the calculation of the total risk-weighted assets, and $\varphi_{\mathrm{h}} \equiv \frac{W_{\mathrm{h}}}{\sqrt{V_{\mathrm{h}}}}$ and $\varphi_{\mathrm{l}} \equiv \frac{W_{\mathrm{l}}}{\sqrt{V_{\mathrm{l}}}}$. We assume that the parameter values are such that all three quantities above are positive.

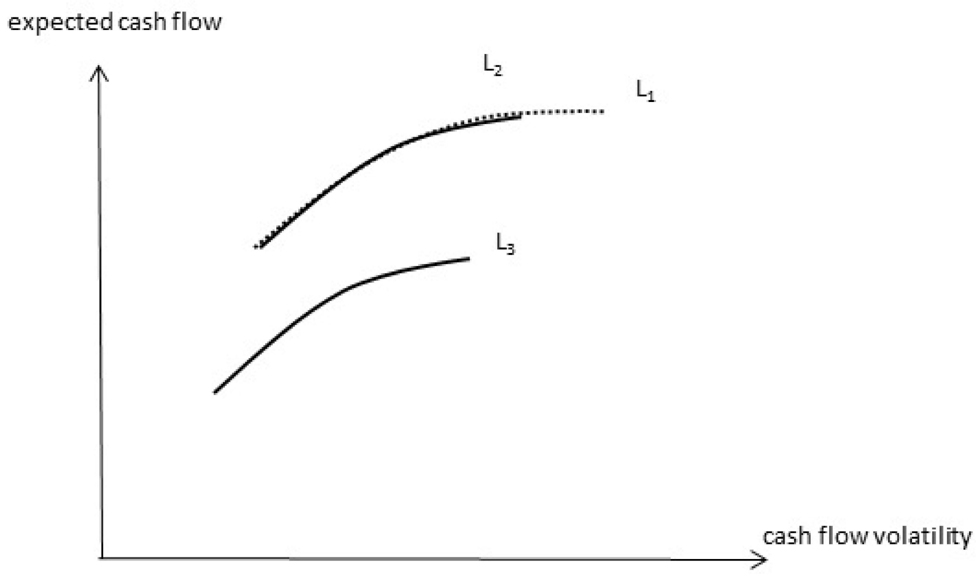
We interpret $\varphi_{\mathrm{h}}$ and $\varphi_{\mathrm{l}}$ as the regulatory cost per asset risk for high- and low-risk assets, respectively. Clearly these costs are important in determining the optimal allocation.

*Impact of a tightening capital requirement*

We then study how the bank reshuffles the portfolio due to new and more stringent capital regulation, such as an increase in the risk-based capital requirement, $k$.

One important effect of a tightening capital requirement on the bank's portfolio is to change its efficient asset investment frontier. For a bank whose capital is not constrained by the current regulation, since it still faces risk-based capital regulation, $W_{\mathrm{h}}A_{\mathrm{h}} + W_{\mathrm{l}}A_{\mathrm{l}} \leq \frac{K}{k}$ (Equation (A.7c)), there are upper limits for the portfolio weights in the risky funds, and hence also for their expected values and variances in terms of cash flows. When the regulator imposes a new regulation and requires the bank to have a higher capital ratio, these upper limits for the risky funds are smaller. Subsequently, those risky funds, whose expected values and variances of cash flows are too high, and that locate far upward and right on the efficient frontier, are now out of reach for the bank. Therefore, the available efficient frontier shrinks from the top and right (illustrated by a move from line $L_1$ to line $L_2$ in Figure 4), and the bank's capital might become constrained by the new regulation.

**Figure 4:** Banks' efficient asset investment frontier




This figure shows banks' efficient asset investment frontier for risky assets. Each curve represents the best possible expected cash flow of a bank's portfolio of assets for its level of risk (cash-flow volatility) under a certain capital regulation rule.

For a bank whose capital is already constrained by regulation, all the risky funds on the efficient frontier reach the upper limits of their expected values and variances of cash flows. Thus, when the regulator imposes a new and more stringent regulation – i.e., a higher $k$ – the bank's efficient frontier falls downward and to the left, since any risky fund's expected value and variance of cash flows decrease: This is illustrated by a move from line $L_2$ to line $L_3$ in Figure 4.

Formally, we can reason as follows around Figure 3. For a bank whose capital is not constrained by regulation, since it still faces the regulatory capital constraint ($W_h A_h + W_l A_l \leq \frac{K}{k}$, Equation (A.7c)), for each risky fund $P$, there is a constraint for the portfolio weight of the low-risk asset, $\omega_l \leq \frac{1}{A_h + A_l}\left(\frac{K}{W_l k} - \frac{W_h}{W_l} A_h\right)$.

Then the expected value and variance of random cash flow for each risky fund $P$ are:

$$\begin{aligned}
E[\widetilde{CF}_P] &= \omega_\mathrm{h}(1-p_\mathrm{h})(1+C_\mathrm{h})+\omega_\mathrm{l}(1-p_\mathrm{l})(1+C_\mathrm{l}) \\
&\leq \frac{1}{A_\mathrm{h}+A_\mathrm{l}}[A_\mathrm{h}(1-p_\mathrm{h})(1+C_\mathrm{h})+\left(\frac{K}{kW_\mathrm{l}}-\frac{W_\mathrm{h}}{W_\mathrm{l}}A_\mathrm{h}\right)(1-p_\mathrm{l})(1+C_\mathrm{l})] \\
&= \omega_\mathrm{h}\left(E[\widetilde{CF}_\mathrm{h}]-\frac{W_\mathrm{h}}{W_\mathrm{l}}E[\widetilde{CF}_\mathrm{l}]\right)+\frac{K}{(A_\mathrm{h}+A_\mathrm{l})kW_\mathrm{l}}E[\widetilde{CF}_\mathrm{l}]\equiv E[\widetilde{CF}_P]_\mathrm{bound}
\end{aligned} \tag{A.23}$$

and

$$\begin{aligned}
\mathrm{Var}[\widetilde{CF}_P] =&\omega_\mathrm{h}^2V_\mathrm{h}+\omega_\mathrm{l}^2V_\mathrm{l}+2\omega_\mathrm{h}\omega_\mathrm{l}\rho\sqrt{V_\mathrm{h}V_\mathrm{l}} \\
\leq&\frac{1}{(A_\mathrm{h}+A_\mathrm{l})^2}[A_\mathrm{h}^2V_\mathrm{h}+\left(\frac{K}{kW_\mathrm{l}}-\frac{W_\mathrm{h}}{W_\mathrm{l}}A_\mathrm{h}\right)^2V_\mathrm{l}+2A_\mathrm{h}(\frac{K}{kW_\mathrm{l}}-\frac{W_\mathrm{h}}{W_\mathrm{l}}A_\mathrm{h})\rho\sqrt{V_\mathrm{h}V_\mathrm{l}}] \\
=&\omega_\mathrm{h}^2\left(V_\mathrm{h}+\frac{W_\mathrm{h}^2}{W_\mathrm{l}^2}V_\mathrm{l}-2\frac{W_\mathrm{h}}{W_\mathrm{l}}\rho\sqrt{V_\mathrm{h}V_\mathrm{l}}\right) \\
&+2\frac{\omega_\mathrm{h}K\sqrt{V_\mathrm{l}}}{(A_\mathrm{h}+A_\mathrm{l})kW_\mathrm{l}}\left(\rho\sqrt{V_\mathrm{h}}-\frac{W_\mathrm{h}}{W_\mathrm{l}}\sqrt{V_\mathrm{l}}\right)+\frac{K^2V_\mathrm{l}}{k^2W_\mathrm{l}^2(V_\mathrm{h}+V_\mathrm{l})^2} \\
\equiv&\mathrm{Var}[\widetilde{CF}_P]_\mathrm{bound},
\end{aligned} \tag{A.24}$$

where $E[\widetilde{CF}_P]_\mathrm{bound}$ and $\mathrm{Var}[\widetilde{CF}_P]_\mathrm{bound}$ are upper limits for $E[\widetilde{CF}_P]$ and $\mathrm{Var}[\widetilde{CF}_P]$, respectively.

Furthermore, as $k$ increases, $E[\widetilde{CF}_P]_\mathrm{bound}$ decreases, and $\mathrm{Var}[\widetilde{CF}_P]_\mathrm{bound}$ decreases since $\frac{\partial\,\mathrm{Var}[\widetilde{CF}_P]_\mathrm{bound}}{\partial k}=\frac{2K\sqrt{V_\mathrm{l}}}{(A_\mathrm{h}+A_\mathrm{l})^2k^2W_\mathrm{l}^2}\left[\left(A_\mathrm{h}W_\mathrm{h}-\frac{K}{k}\right)\sqrt{V_\mathrm{l}}-A_\mathrm{h}\rho W_\mathrm{l}\sqrt{V_\mathrm{h}}\right]<0$ (as $A_\mathrm{h}W_\mathrm{h}\leq\frac{K}{k}$ and $\rho>0$).

When the regulator imposes a new and more stringent capital requirement – i.e., $k$ increases – for some risky funds, the expected value and variance of their random cash flows are higher than the respective upper limits. Hence, these risky funds are out of reach and the efficient frontier for the bank shrinks from the top and right.

For a bank whose capital is already constrained by regulation, the expected values and variances of cash flows of all the risky funds on its efficient frontier reach their respective upper limits, which decrease as $k$ increases. Therefore, the new regulation forces the efficient frontier to move downward and to the left.

How will the bank whose capital is already constrained by the current regulation reshuffle its optimal portfolio due to the new regulation? The following proposition is derived from the first derivatives of the optimal portfolio weight of high-risk assets in the risky fund, $\omega^*_{\mathrm{bh}} \equiv \frac{A^*_{\mathrm{bh}}}{A^*_{\mathrm{bh}} + A^*_{\mathrm{bl}}}$ and of the optimal investment in the risk-free fund, $G^*_{\mathrm{b}}$, with respect to $k$.

It transpires that there are two key parameters determining the bank's choice: $\vartheta_{\mathrm{h}} \equiv [(1-p_{\mathrm{h}})(1+C_{\mathrm{h}}) - (1+r_{\mathrm{f}})]/W_{\mathrm{h}}$ and $\vartheta_{\mathrm{l}} \equiv [(1-p_{\mathrm{l}})(1+C_{\mathrm{l}}) - (1+r_{\mathrm{f}})]/W_{\mathrm{l}}$. $\vartheta_{\mathrm{h}}$ and $\vartheta_{\mathrm{l}}$ measure expected excess cash flows per capital cost due to the regulation for high- and low-risk assets, respectively. We call $\vartheta_{\mathrm{h}}$ and $\vartheta_{\mathrm{l}}$ the reward-to-regulatory-cost ratios, and we note that they can be written as $\vartheta_h = (\mu_h - r_f)/W_h$ and $\vartheta_l = (\mu_l - r_f)/W_l$, where $\mu_h \equiv (1-p_{\mathrm{h}})(1+C_{\mathrm{h}}) - 1$ is the expected net return on the high-risk asset and $\mu_l \equiv (1-p_{\mathrm{l}})(1+C_{\mathrm{l}}) - 1$ is the expected expected net return on the low-risk asset.

**Proposition 3.** *When the bank's capital is constrained by regulation and the regulator imposes a new and more stringent regulation with a higher capital requirement k,*

(a) *within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, if $\vartheta_{\mathrm{h}} > \vartheta_{\mathrm{l}}$ ($\vartheta_{\mathrm{h}} < \vartheta_{\mathrm{l}}$), and*

(b) *the bank invests more in the risk-free fund, ceteris paribus, given that $\vartheta_{\mathrm{h}} \geq \vartheta_{\mathrm{l}}$ and $\rho\varphi_{\mathrm{l}} \geq \varphi_{\mathrm{h}}$.*

*Proof: (a) When the bank's capital is constrained by regulation and the regulator imposes a new and more stringent regulation with a higher capital requirement, k, within the risky fund, the bank invests proportionally more (less) in high-risk assets, ceteris paribus, given that $\vartheta_{\mathrm{h}} > \vartheta_{\mathrm{l}}$ ($\vartheta_{\mathrm{h}} < \vartheta_{\mathrm{l}}$).*

$$\frac{\partial \omega_{\mathrm{bh}}^{*}}{\partial k} = \frac{\partial \dfrac{A_{\mathrm{bh}}^{*}}{A_{\mathrm{bh}}^{*}+A_{\mathrm{bl}}^{*}}}{\partial k} = \frac{A_{\mathrm{bh}}^{*2} K \sqrt{V_{\mathrm{h}}}(a-B)(\varphi_{\mathrm{l}} SR_{\mathrm{h}} - \varphi_{\mathrm{h}} SR_{\mathrm{l}})[(\varphi_{\mathrm{l}} SR_{\mathrm{h}} - \varphi_{\mathrm{h}} SR_{\mathrm{l}})^2 + (\varphi_{\mathrm{h}}^2 - 2\rho\varphi_{\mathrm{h}}\varphi_{\mathrm{l}} + \varphi_{\mathrm{l}}^2)]}{(A_{\mathrm{bh}}^{*}+A_{\mathrm{bl}}^{*})^2 \sqrt{V_{\mathrm{l}}}\{k(a-B)\varphi_{\mathrm{l}}(\varphi_{\mathrm{l}} SR_{\mathrm{h}} - \varphi_{\mathrm{h}} SR_{\mathrm{l}}) - K[SR_{\mathrm{l}}(\varphi_{\mathrm{l}} SR_{\mathrm{h}} - \varphi_{\mathrm{h}} SR_{\mathrm{l}}) + (\rho\varphi_{\mathrm{l}} - \varphi_{\mathrm{h}})]\}^2} > 0, \tag{A.25}$$

given that $\varphi_{\mathrm{l}} SR_{\mathrm{h}} > \varphi_{\mathrm{h}} SR_{\mathrm{l}}$ – i.e., $\vartheta_{\mathrm{h}} > \vartheta_{\mathrm{l}}$ – since $a > B$ and $\varphi_{\mathrm{h}}^2 - 2\rho\varphi_{\mathrm{h}}\varphi_{\mathrm{l}} + \varphi_{\mathrm{l}}^2 = (\varphi_{\mathrm{h}} - \varphi_{\mathrm{l}})^2 + 2(1-\rho)\varphi_{\mathrm{h}}\varphi_{\mathrm{l}} > 0$. The sign of the above expression is reversed if instead $\vartheta_{\mathrm{h}} < \vartheta_{\mathrm{l}}$.

*(b) When the bank's capital is constrained by regulation and the regulator imposes a new and more stringent regulation with a higher capital requirement, k, the bank invests more in the risk-free fund, ceteris paribus, given that $\vartheta_{\mathrm{h}} \geq \vartheta_{\mathrm{l}}$ and $\rho\varphi_{\mathrm{l}} \geq \varphi_{\mathrm{h}}$.*

$$\begin{aligned} \frac{\partial G_{\mathrm{b}}^{*}}{\partial k} &= \frac{K}{k^2 W_{\mathrm{l}}} + \frac{W_{\mathrm{h}} - W_{\mathrm{l}}}{W_{\mathrm{l}}} \frac{\partial A_{\mathrm{bh}}^{*}}{\partial k} \\ &= \frac{K}{k^2 W_{\mathrm{l}}} + \frac{W_{\mathrm{h}} - W_{\mathrm{l}}}{W_{\mathrm{l}}} \frac{K\{SR_{\mathrm{l}}(\varphi_{\mathrm{l}} SR_{\mathrm{h}} - \varphi_{\mathrm{h}} SR_{\mathrm{l}}) + (\rho\varphi_{\mathrm{l}} - \varphi_{\mathrm{h}})\}}{k^2(\varphi_{\mathrm{h}}^2 - 2\rho\varphi_{\mathrm{h}}\varphi_{\mathrm{l}} + \varphi_{\mathrm{l}}^2)\sqrt{V_{\mathrm{h}}} + k^2(\varphi_{\mathrm{l}} SR_{\mathrm{h}} - \varphi_{\mathrm{h}} SR_{\mathrm{l}})^2\sqrt{V_{\mathrm{h}}}} > 0, \end{aligned} \tag{A.26}$$

given that $\varphi_{\mathrm{l}} SR_{\mathrm{h}} \geq \varphi_{\mathrm{h}} SR_{\mathrm{l}}$ – i.e., $\vartheta_{\mathrm{h}} \geq \vartheta_{\mathrm{l}}$ – and $\rho\varphi_{\mathrm{l}} \geq \varphi_{\mathrm{h}}$, since $W_{\mathrm{h}} > W_{\mathrm{l}}$. □

Statement (a) tells us that, if the bank whose capital is already constrained by the regulation is required to have a higher capital ratio, it reshuffles the risky fund and invests proportionally more in the asset with the higher reward-to-regulatory-cost ratio. In contrast with the Sharpe ratio, which is a reward-to-variability ratio, the denominator of the reward-to-regulatory-cost ratio is the risk weighting assigned to that type of risky asset by the regulator. Therefore, if the risk weightings are not consistent with the assets' cash-flow variances, which are measures of risk in this model, we could predict that there would be opportunities for regulatory arbitrage.

Statement (b) shows that when the bank is required to have a higher capital ratio, it invests more in the risk-free fund, if the regulatory cost per asset risk for a low-risk asset is much higher than that for a high-risk asset whose valuation (reward-to-regulatory-cost ratio) is not lower. That is, the risk-free fund mitigates the risk of a higher regulatory cost, when low-risk assets are too costly to provide such mitigation.

**Appendix B: Estimating default probability and default correlation**

We adopt the method of calculating average cumulative default rates with adjustment for rating withdrawals used by Moody's, as demonstrated by Cantor and Hamilton (2007).

A cumulative default rate for an investment horizon of length $T$, denoted as $D(T)$ is formulated as

$$\begin{aligned} D_y(T) &= d_y(1) + d_y(2)[1-d_y(1)] + d_y(3)[(1-d_y(1))(1-d_y(2))] + \dots \\ &\quad + d_y(T)(\prod_{t=1}^{T-1}[1-d_y(t)]) = 1 - \prod_{t=1}^{T}[1-d_y(t)], \end{aligned} \tag{B.1}$$

where $d_y(t)$ is the marginal default rate in the time interval $t$[21] for a cohort of issuers formed on date $y$ holding a certain rating and calculated as $d_y(t) = \frac{x_y(t)}{n_y(t)}$, where $x$ is the number of defaults and $n$ is the effective size of the cohort adjusted for rating withdrawals. As displayed, the cumulative default rate is essentially a discrete-time approximation of the nonparametric continuous-time–hazard-rate approach and a conditional probability.

We adopt average cumulative default rates, where the average is taken over many cohort periods, to estimate default probabilities in our study. The average cumulative default rate for an investment horizon of length $T$, denoted as $\bar{D}(T)$, is derived from the weighted average marginal default rates, $\bar{d}(t)$, where the average is taken over all the available cohort marginal default rates in the historical

[21]For example, in the first period after the formation of a cohort, $t = 1$; in the second period after the formation of a cohort, $t = 2$; etc..

data set $Y$.

Then $\bar{D}(T) = 1 - \prod_{t=1}^{T}[1 - \bar{d}(t)]$, where $\bar{d}(t) = \dfrac{\sum_{y \in Y} x_y(t)}{\sum_{y \in Y} n_y(t)}$.

As we estimate the default correlations, we modify the above methodology accordingly.

The pair-wise default probability, for one corporation with rating 1 and another with rating 2 in the time interval $t$, is $\dfrac{x_y^1(t)x_y^2(t)}{n_y^1(t)n_y^2(t)}$, where $x_y^1$ and $x_y^2$ are the numbers of defaults for cohorts of issuers holding rating 1 and 2 formed on date $y$ respectively, and $n_y^1$ and $n_y^2$ are the corresponding effective sizes of the cohorts. Then, the average pair-wise default rate in the time interval $t$ over all available cohorts is $\bar{d_{12}}(t) = \dfrac{\sum_{y \in Y} x_y^1(t)x_y^2(t)}{\sum_{y \in Y} n_y^1(t)n_y^2(t)}$.

Hence, we could estimate the default correlation for the investment horizon of length $T$ by an average over all available marginal default correlations in the data set $Y$:

$$\bar{\rho}_{12}(T) = \frac{1}{T-1}\sum_{t=1}^{T-1}\rho_{12}(t) = \frac{1}{T-1}\sum_{t=1}^{T-1}\frac{\bar{d_{12}}(t) - \bar{d}_1(t)\bar{d}_2(t)}{\sqrt{\bar{d}_1(t)[1-\bar{d}_1(t)]\bar{d}_2(t)[1-\bar{d}_2(t)]}} \tag{B.2}$$

where $\rho_{12}(t)$ is the marginal default correlation in the time interval $t$.[22]

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[22] $\rho_{12}(T)$ is excluded in the calculation of $\bar{\rho}_{12}(T)$ because there is only one observation at $T$ resulting from one cohort left with a certain rating, which results in zero correlation.

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**Table 2:** Proportion of high-risk assets in the risky fund – benchmark model

| | (1) excluding the crisis period | | (2) excluding the crisis period | | (3) excluding the crisis period | | (4) including the crisis period | |
|---|---|---|---|---|---|---|---|---|
| VARIABLES | Coefficient | Marginal Effect | Coefficient | Marginal Effect | Coefficient | Marginal Effect | Coefficient | Marginal Effect |
| Payoff of low-risk assets | 0.65 | 0.16 | -0.52 | -0.12 | -0.94 | -0.22 | -0.51 | -0.12 |
| | (0.55) | (0.13) | (0.91) | (0.21) | (0.92) | (0.22) | (0.89) | (0.21) |
| Payoff of high-risk assets | 0.96*** | 0.23*** | 0.75*** | 0.18*** | 0.98*** | 0.23*** | 0.79*** | 0.19*** |
| | (0.19) | (0.046) | (0.28) | (0.065) | (0.28) | (0.067) | (0.27) | (0.062) |
| Default prob. of low-risk assets | 60.6*** | 14.7*** | 30.7*** | 7.26*** | 25.9*** | 6.12*** | 22.1*** | 5.19*** |
| | (3.94) | (0.94) | (6.05) | (1.43) | (5.48) | (1.29) | (4.70) | (1.10) |
| Default prob. of high-risk assets | -8.46*** | -2.05*** | -4.47*** | -1.06*** | -3.47*** | -0.82*** | -3.24*** | -0.76*** |
| | (0.55) | (0.13) | (0.95) | (0.23) | (0.91) | (0.21) | (0.89) | (0.21) |
| Default correlation | -2.21*** | -0.54*** | -1.37** | -0.33** | -1.24** | -0.29** | -1.42*** | -0.33*** |
| | (0.27) | (0.065) | (0.54) | (0.13) | (0.54) | (0.13) | (0.35) | (0.082) |
| Basel II | | | | | 0.042*** | 0.010*** | 0.032** | 0.0075** |
| | | | | | (0.013) | (0.0032) | (0.013) | (0.0031) |
| Equity ratio | | | -1.83*** | -0.43*** | -1.82*** | -0.43*** | -1.50** | -0.35** |
| | | | (0.69) | (0.16) | (0.69) | (0.16) | (0.67) | (0.16) |
| Ln(size) | | | 0.048 | 0.011 | 0.044 | 0.010 | 0.053 | 0.012 |
| | | | (0.038) | (0.0089) | (0.038) | (0.0089) | (0.037) | (0.0086) |
| Liquid assets | | | -2.09*** | -0.49*** | -2.08*** | -0.49*** | -2.27*** | -0.53*** |
| | | | (0.15) | (0.037) | (0.15) | (0.037) | (0.14) | (0.033) |
| Return on assets | | | -1.75*** | -0.41*** | -1.66** | -0.39** | -1.98*** | -0.47*** |
| | | | (0.68) | (0.16) | (0.67) | (0.16) | (0.63) | (0.15) |
| Nonperforming assets | | | 0.18 | 0.044 | 0.15 | 0.036 | 0.55 | 0.13 |
| | | | (0.64) | (0.15) | (0.64) | (0.15) | (0.52) | (0.12) |
| Cost-to-income ratio | | | -0.15** | -0.035** | -0.15** | -0.035** | -0.13*** | -0.032*** |
| | | | (0.063) | (0.015) | (0.062) | (0.015) | (0.051) | (0.012) |
| Noninterest income | | | -0.18 | -0.044 | -0.19 | -0.045 | -0.25** | -0.059** |
| | | | (0.13) | (0.031) | (0.13) | (0.031) | (0.11) | (0.027) |
| Constant | 0.39*** | | 0.98*** | | 0.92*** | | 0.93*** | |
| | (0.023) | | (0.20) | | (0.20) | | (0.20) | |
| Observations | 51,489 | | 16,162 | | 16,162 | | 18,607 | |
| Number of banks | 1720 | | 596 | | 596 | | 597 | |
| Pseudo R-squared | 0.018 | | 0.32 | | 0.32 | | 0.34 | |
| Chi2 | 400 | | 774 | | 773 | | 962 | |
| Prob>Chi2 | 0 | | 0 | | 0 | | 0 | |

*Payoff*, *Default probability*, and *Default correlation* are based on the average macro credit information on the risky assets from our estimation. *Basel II* is a dummy variable taking on a value of one starting from the third quarter of 2004, after the introduction of Basel II by the Basel Committee. The bank-level controls are *Equity ratio* (the ratio of total equity to total assets), $ln(Size)$ (natual logrithm of total assets in billions of dollars), *Liquid assets* (the ratio of liquid assets to total assets), *Return on assets*, *Nonperforming assets* (the ratio of nonperforming assets to total assets), *Cost-to-income ratio*, and *Noninterest income* (the ratio of the absolute value of noninterest income to the sums of the absolute values of noninterest and interest income). The estimated payoffs, default probabilities, and default correlation (in percentage) are valued at the beginning of each quarter. A generalized linear model with binomial distribution and logit link function is used for estimation, and the time-series averages of bank-level control variables are added to account for correlation between unobserved heteroscedasticity and the explanatory variables, as proposed by Papke and Wooldridge (2008). For brevity, we do not report the coefficients for the time averages. In parentheses are standard errors which are robust to heteroskedasticity and serial dependence. $^{*}$, $^{**}$, and $^{***}$ denote significance at the 10%, 5%, and 1% confidence levels, respectively.

**Table 3:** Actual allocation vs. prediction during Basel II

| Variable | Obs. | Mean | Std. Dev. | Min. | Max. |
|---|---|---|---|---|---|
| Proportion of high-risk assets | 5 | 0.810 | 0.069 | 0.745 | 0.886 |
| Predicted proportion of high-risk assets | 5 | 0.727 | 0.065 | 0.654 | 0.800 |
| Difference of actual allocation with the prediction | 5 | 0.083 | 0.027 | 0.040 | 0.113 |
| Risk-based capital ratio | 5 | 7 836 | .111 | 7.67 | 7.95 |

We estimate the model in specification (2) in Table 2 for banks that are unconstrained during the Basel I period, and use it to predict the proportion of high-risk assets after the implementation of Basel II. This table displays the banks that become constrained during Basel II.

**Table 4:** Proportion of high-risk assets in the risky fund – dynamic model

| | (1) | | (2) | | (3) | |
|---|---|---|---|---|---|---|
| VARIABLES | Coefficient | Marginal Effect | Basel I | Basel I Marginal | Basel II | Basel II Marginal |
| Proportion of high-risk assets-L1 | 4.14*** | 0.96*** | 3.71*** | 0.87*** | 4.50*** | 1.03*** |
| | (0.073) | (0.017) | (0.10) | (0.025) | (0.084) | (0.019) |
| Proportion of high-risk assets-L2 | 0.23*** | 0.054*** | 0.47*** | 0.11*** | -0.041 | -0.0093 |
| | (0.070) | (0.016) | (0.095) | (0.022) | (0.092) | (0.021) |
| Proportion of high-risk assets-L3 | 0.016 | 0.0038 | 0.10 | 0.024 | -0.031 | -0.0072 |
| | (0.053) | (0.012) | (0.063) | (0.015) | (0.079) | (0.018) |
| Payoff of low-risk assets | 1.03*** | 0.24*** | -5.27 | -1.24 | 0.62 | 0.14 |
| | (0.25) | (0.058) | (3.61) | (0.85) | (0.95) | (0.22) |
| Payoff of low-risk assets-L1 | -0.31 | -0.073 | -10.4 | -2.45 | -3.67** | -0.84** |
| | (0.29) | (0.068) | (7.32) | (1.73) | (1.87) | (0.43) |
| Payoff of low-risk assets-L2 | -0.49* | -0.11* | -9.04 | -2.13 | -1.27 | -0.29 |
| | (0.27) | (0.063) | (6.13) | (1.45) | (0.80) | (0.18) |
| Payoff of low-risk assets-L3 | 0.69*** | 0.16*** | -5.11 | -1.21 | -0.73 | -0.17 |
| | (0.25) | (0.058) | (4.34) | (1.02) | (0.59) | (0.14) |
| Payoff of high-risk assets | -0.44*** | -0.10*** | 4.30 | 1.01 | -1.16*** | -0.27*** |
| | (0.13) | (0.029) | (4.64) | (1.09) | (0.42) | (0.096) |
| Payoff of high-risk assets-L1 | -0.0046 | -0.0011 | 7.04 | 1.66 | 0.57 | 0.13 |
| | (0.16) | (0.036) | (5.87) | (1.39) | (0.70) | (0.16) |
| Payoff of high-risk assets-L2 | -0.61*** | -0.14*** | 10.1 | 2.38 | 0.041 | 0.0094 |
| | (0.14) | (0.031) | (8.05) | (1.90) | (0.49) | (0.11) |
| Payoff of high-risk assets-L3 | 0.11 | 0.025 | 6.40 | 1.51 | 1.74*** | 0.40*** |
| | (0.14) | (0.033) | (4.09) | (0.96) | (0.57) | (0.13) |
| Default prob. of low-risk assets | -4.93*** | -1.15*** | 39.6 | 9.34 | 10.4 | 2.38 |
| | (1.70) | (0.39) | (44.6) | (10.5) | (6.54) | (1.50) |
| Default prob. of low-risk assets-L1 | -1.88 | -0.44 | 14.0 | 3.31 | 16.7** | 3.84** |
| | (1.64) | (0.38) | (21.6) | (5.10) | (6.56) | (1.50) |
| Default prob. of low-risk assets-L2 | 2.38 | 0.55 | 59.5 | 14.0 | 9.95*** | 2.28*** |
| | (1.67) | (0.39) | (54.8) | (12.9) | (3.67) | (0.84) |
| Default prob. of low-risk assets-L3 | -4.63*** | -1.08*** | 164 | 38.6 | 5.83 | 1.34 |
| | (1.38) | (0.32) | (143) | (33.8) | (4.88) | (1.12) |
| Default prob. of high-risk assets | 2.34*** | 0.54*** | -22.1 | -5.21 | -0.76 | -0.17 |
| | (0.50) | (0.12) | (17.2) | (4.06) | (1.63) | (0.37) |
| Default prob. of high-risk assets-L1 | -0.42 | -0.098 | -29.0 | -6.85 | -1.17 | -0.27 |
| | (0.55) | (0.13) | (20.9) | (4.92) | (1.13) | (0.26) |
| Default prob. of high-risk assets-L2 | -1.75*** | -0.41*** | 7.62 | 1.80 | -4.14*** | -0.95*** |
| | (0.44) | (0.10) | (6.24) | (1.47) | (1.48) | (0.34) |
| Default prob. of high-risk assets-L3 | 0.92** | 0.21** | 13.4* | 3.16* | -0.093 | -0.021 |
| | (0.36) | (0.084) | (7.63) | (1.80) | (0.92) | (0.21) |
| Default correlation | -0.19 | -0.044 | -32.8 | -7.74 | -0.94*** | -0.22*** |
| | (0.14) | (0.032) | (26.4) | (6.22) | (0.34) | (0.078) |
| Default correlation-L1 | 0.20 | 0.047 | -14.5 | -3.41 | -0.77 | -0.18 |
| | (0.16) | (0.038) | (11.4) | (2.68) | (0.49) | (0.11) |
| Default correlation-L2 | 0.13 | 0.031 | 12.7 | 3.01 | -1.32** | -0.30** |
| | (0.14) | (0.033) | (13.3) | (3.13) | (0.67) | (0.15) |
| Default correlation-L3 | -0.18 | -0.043 | 2.08 | 0.49 | 0.65 | 0.15 |

Continued on next page

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**Table 4:** Proportion of high-risk assets in the risky fund – dynamic model

| | (1) | | (2) | | (3) | |
|---|---|---|---|---|---|---|
| VARIABLES | Coefficient | Marginal Effect | Basel I | Basel I Marginal | Basel II | Basel II Marginal |
| | (0.14) | (0.032) | (1.89) | (0.45) | (0.44) | (0.10) |
| Crisis | -0.0098** | -0.0023** | 0.035 | 0.0082 | 0.12** | 0.027** |
| | (0.0040) | (0.00093) | (0.032) | (0.0075) | (0.058) | (0.013) |
| *CAMELS variables* | YES | YES | YES | YES | YES | YES |
| | | | | | | |
| Constant | -2.23*** | | -2.32*** | | -2.23*** | |
| | (0.032) | | (0.32) | | (0.052) | |
| Observations | 17,256 | | 8,117 | | 9,139 | |
| Number of banks | 590 | | 495 | | 486 | |
| Pseudo R-squared | 0.96 | | 0.97 | | 0.96 | |
| Chi2 | 36445 | | 33217 | | 27982 | |
| Prob>Chi2 | 0 | | 0 | | 0 | |

*Payoff*, *Default probability*, and *Default correlation* are based on the average macro credit information on the risky assets from our estimation. *Crisis* is a dummy variable taking on a value of one during the financial crisis period, from 2007Q4 to 2009Q2, as defined by NBER. The bank-level controls (*CAMELS variables*) are *Equity ratio* (the ratio of total equity to total assets), $ln(Size)$ (natual logrithm of total assets in billions of dollars), *Liquid assets* (the ratio of liquid assets to total assets), *Return on assets*, *Nonperforming assets* (the ratio of nonperforming assets to total assets), *Cost-to-income ratio*, and *Noninterest income* (the ratio of the absolute value of noninterest income to the sums of the absolute values of noninterest and interest income). The estimated payoffs, default probabilities, and default correlation (in percentage) are valued at the beginning of each quarter. A generalized linear model with binomial distribution and logit link function is used for estimation, and the time-series averages of bank-level control variables are added to account for correlation between unobserved heteroscedasticity and the explanatory variables, as proposed by Papke and Wooldridge (2008). For brevity, we do not report the coefficients for the time averages. In parentheses are standard errors which are robust to heteroscedasticity and serial dependence. $^{*}$, $^{**}$, and $^{***}$ denote significance at the 10%, 5%, and 1% confidence levels, respectively.